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**A LIKELIHOOD STORY: THE THEORY  
OF LEGAL FACT-FINDING**

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*Are racial stereotypes a proper basis for legal fact-finding? What about gender stereotypes, sincerely believed by the fact-finder and informed by the fact-finder's life experience? What about population averages: if people of a certain gender, education level, and past criminal history exhibit a statistically greater incidence of violent behavior than the population overall, is this evidence that a given person within this class did act violently on a particular occasion?*

*The intuitive answer is that none of these feel like proper bases on which fact-finders should be deciding cases. But why not? Nothing in traditional probability or belief-based theories of fact-finding justifies excluding any of these inferences. Maybe intuition goes astray here. Or maybe something about the traditional theory of fact-finding is wrong. Arguing the latter, this article proposes a new theory of fact-finding. In contrast to historic probability and belief-based theories, this paper suggests that idealized fact-finding is an application of likelihood reasoning—the statistical analog of*

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*the ancient legal concept of the “weight of evidence” and the formal analog of modern descriptions of legal fact-finding as a process of comparing the relative plausibility of competing factual stories on the evidence.*

*This likelihood theory marks a fundamental change in our understanding of fact-finding, with equally fundamental implications for practice and procedure. The theory simplifies fact-finding, describing every burden of persuasion as an application of the same reasoning principle. It harmonizes recent scholarship on fact-finding, showing that work on the cognitive processes of fact-finders can be formalized in a comprehensive and coherent theory of the ideal fact-finding process. It explains evidentiary mores, justifying hostility to naked statistical evidence, for example. And it provides new insights into the effects of subjective beliefs on fact-finding, showing not only the harm that results from asking fact-finders to decide cases based on their personal beliefs about the facts, but also the way forward in reorienting fact-finding away from prejudice, bias, and subjective beliefs, and toward the firmer ground of the evidence itself.*

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## INTRODUCTION

In oral arguments for the recent Supreme Court case of *Peña Rodriguez v. Colorado*, Justice Sotomayor voiced a rhetorical but serious question: “Don’t we want deliberations on evidence and not deliberations on someone’s stereotypes and feelings about the race of a defendant?”<sup>1</sup> The answer is obviously, “Yes!” But why was the question even asked? How could it be that over two hundred years into the history of our country, a justice of the highest court of the land would have the occasion to ask a question with so startlingly obvious an answer?

Other questions only raised the level of alarm. Justice Alito asked whether juror discrimination on the basis of a party’s religion differed from racial discrimination.<sup>2</sup> The Chief Justice pushed this point: “You think it’s odious to have [jurors] discriminat[e] against someone because he’s a Muslim or practices Islamic faith? You’re saying, . . . I know how Muslims behave; he committed this crime.”<sup>3</sup> Justice Ginsburg asked whether gender discrimination in an automotive case would be different: “Suppose [a juror] says, what do you expect of women drivers? Women shouldn’t be allowed to drive cars. Every woman I know is a terrible driver.”<sup>4</sup> Again, these questions shock only because their answers are so obvious. None of these inferences are different from one another in substance; all of them are offensive to our sense of justice.<sup>5</sup> Take, for example, the belief that women are bad drivers. To allow this belief to color fact-finding—and thus to allow every woman defendant to

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1. Transcript of Oral Argument at 40:5–7, *Peña-Rodriguez v. Colorado*, 137 S. Ct. 855 (2017) (No. 15-606).

2. *Id.* at 5:10–13.

3. *Id.* at 6:16–21.

4. *Id.* at 12:1–5.

5. In deciding this case, the Court distinguished race-based discrimination from other forms of prejudice on grounds such as the history of race relations and race-specific jurisprudence in this country. *Peña-Rodriguez*, 137 S. Ct. at 868–69. While I fully accept that racial discrimination produces special harms, distinguishing it from other forms of prejudice only defers the fundamental question: when, if ever, should prior prejudice be permitted in fact-finding—and why?

be found liable by what is effectively a lower burden of persuasion than would apply to an otherwise identical male defendant—is anathema to our most basic concepts of fairness and justice.

Again, why were these questions asked at all? The case involved an effort to impeach a jury’s verdict, and the issue facing the Court was whether jurors should be allowed to testify about racially biased deliberation statements<sup>6</sup> in spite of the usual prohibition on juror testimony about deliberations during inquiries into the validity of the verdict.<sup>7</sup> But the most salient feature of the oral arguments was the close availability of so many cringingly realistic examples of biased fact-finding. And while each of the justices appeared to agree that these examples involved unfairness, looming uncomfortably over the proceeding was a deeper and unanswered question: *why* are these inferences unfair?

Uncertainty over this question may go a long way in explaining the doubtless prevalence of biased fact-finding today. Jurors are typically instructed to use common sense and their personal experiences in their deliberations,<sup>8</sup> but where personal experience translates into prejudicial bias is a detail that goes conspicuously unexplored.<sup>9</sup> This failure to distinguish evidential weight from prejudicial bias is attributable to basic un-

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6. *Peña-Rodriguez*, 137 S. Ct. at 862.

7. *See, e.g.*, FED. R. EVID. 606(b)(1) (providing, subject to limited exceptions, that in “an inquiry into the validity of a verdict or indictment, a juror may not testify about any statement made or incident that occurred during the jury’s deliberations”). Many state rules of evidence include similar prohibitions.

8. *See, e.g.*, THIRD CIRCUIT MODEL CIVIL JURY INSTRUCTIONS § 1.5 (2017), [http://www.ca3.uscourts.gov/sites/ca3/files/1\\_Chaps\\_1\\_2\\_3\\_2017\\_October.pdf](http://www.ca3.uscourts.gov/sites/ca3/files/1_Chaps_1_2_3_2017_October.pdf) [<https://perma.cc/P9EX-WMCT>] (“You should use your common sense in weighing the evidence. Consider it in light of your everyday experience with people and events, and give it whatever weight you believe it deserves.”); THIRD CIRCUIT MODEL CRIMINAL JURY INSTRUCTIONS § 1.08 (2012), [http://www.ca3.uscourts.gov/sites/ca3/files/2012%20Chapter%201\\_0.pdf](http://www.ca3.uscourts.gov/sites/ca3/files/2012%20Chapter%201_0.pdf) [<https://perma.cc/KW5M-A48V>] (same); SEVENTH CIRCUIT FEDERAL CIVIL JURY INSTRUCTIONS § 1.11 (2017), [http://www.ca7.uscourts.gov/pattern-jury-instructions/7th\\_cir\\_civil\\_instructions.pdf](http://www.ca7.uscourts.gov/pattern-jury-instructions/7th_cir_civil_instructions.pdf) [<https://perma.cc/Z49B-R2NK>] (“You should use common sense in weighing the evidence and consider the evidence in light of your own observations in life.”); SEVENTH CIRCUIT PATTERN CRIMINAL JURY INSTRUCTIONS § 2.02 (2012), [http://www.ca7.uscourts.gov/pattern-jury-instructions/7th\\_criminal\\_jury\\_instr.pdf](http://www.ca7.uscourts.gov/pattern-jury-instructions/7th_criminal_jury_instr.pdf) [<https://perma.cc/WVK5-V5MV>] (similar).

9. *See United States v. Jones*, 808 F.2d 561, 568 (7th Cir. 1986) (concluding, without analysis or citation, that instructions to use common sense in deliberations “[do] not . . . invite a jury member to consider the evidence in light of personally-held . . . stereotypes or prejudices.”).

certainties about how the fact-finding process is meant to work.<sup>10</sup> Yet recent efforts to clarify the theory of fact-finding have only made things worse.<sup>11</sup> The now-popular interpretation of fact-finding in terms of Bayesian probability analysis does nothing to clarify the line between personal experience and bias. If anything, it requires fact-finders to bring their personal biases and prejudices into the courtroom.

To see the connection between hateful bias and abstract probability, consider the following puzzle. Suppose I have two cups, each containing six marbles. The cups are indistinguishable from the outside. One of them has five white marbles and one red (call this the *white cup*); the other has five red marbles and one white (call this the *red cup*). Now consider two simple questions. First, suppose you were to draw a marble at random from the white cup. What is the probability that you would draw a white marble? Second, suppose I inspect both cups and choose one of them; then, without revealing to you which cup I chose, allow you to randomly draw a marble from this cup—which comes out white. Given this observation, what is the probability that I have chosen the white cup?

The answer to the first question is easy: the probability of drawing a white marble from the white cup is  $5/6$  (about 83%). The answer to the second question is harder. Most people would comfortably guess that I chose the white cup in the second question, but the exact probability of that choice is not obvious. While some may recall that there is a rule of probability for this calculation (Bayes' Theorem),<sup>12</sup> few have an intuitive

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10. See generally J.P. McBaine, *Burden of Proof: Degrees of Belief*, 32 CALIF. L. REV. 242 (1944) (noting widespread uncertainty about what degree of persuasion is needed to justify relief in different situations, what measure of persuasion should be the basis for findings of fact, and how jurors should be instructed to go about finding facts).

11. Theoretical work on fact-finding seeks formal answers to the types of questions raised by McBaine, *supra* note 10. See, e.g., Chris William Sanchirico, *Evidence: Theoretical Models*, in 8 ENCYCLOPEDIA OF LAW AND ECONOMICS 203, 207–13 (Gerrit De Geest ed., 2017), <https://doi.org/10.4337/9781782540472.00014> [<https://perma.cc/DJH8-4VUD>] (surveying the Bayesian probability model of fact-finding in which fact-finder beliefs are iteratively updated to reflect observed evidence, and these beliefs are used as the basis for finding the facts of a case).

12. Summary exposition of Bayes' Theorem can be found in any introductory text on probability and statistics. E.g., LEE J. BAIN & MAX ENGELHARDT, INTRODUCTION TO PROBABILITY & MATHEMATICAL STATISTICS 22–27 (2d ed. 2000). Though typically attributed to Thomas Bayes, the origins of the theorem are murky. See generally Stephen M. Stigler, *Who Discovered Bayes's Theorem?*, 37 AM. STATISTICIAN 290 (1983).

grasp of what exactly it entails.

Even with the formula in hand, the situation is not so simple. There was indeed a  $5/6$  probability of drawing a white marble in the first question. But in the second question, it does not make much sense to talk about the *probability* of my choice at all. For one thing, this is not a future event with value still to be realized: I've already chosen the cup. That historic choice is now a fact in the world, not a random variable. The probability that I chose the white cup is—in a literal sense—either zero or one. The connection to legal fact-finding is transparent. What is the probability that the defendant fired the gun? Zero or one. It is odd indeed that the Federal Rules of Evidence talk of relevant evidence as having “any tendency to make a fact more or less *probable* than it would be without the evidence,”<sup>13</sup> when in most cases there is nothing probabilistic about the material facts at all.

The complexity goes further. Even if you did persist in trying to compute the “probability” of my choice using Bayes' Theorem, you would have found that you needed to know something I didn't tell you: my latent, or prior, probability of choosing the white cup over the red cup. But I gave no prior probability for this choice because it, too, was not random. I chose one of the cups by conscious, willful act. There was nothing probabilistic about my choice. Undeterred, you might think that the way to solve this difficulty is to simply assume a probability distribution for my choice of cup: you might guess, for example, that there was a 50% chance that I would choose the white cup.

Return again to legal fact-finding. How would this translate? If the facts at issue are historic acts and conscious decisions—which, like my choice of cup, are not truly governed by prior probabilities—then on what basis would you even try to guess a prior probability distribution? What is the prior probability that someone would commit a crime, or fail to exercise due care? Perhaps these questions would be left to the fact-finder to be answered by personal experience and belief. But you already see the problem. What is the probability that a Muslim man committed a crime? What is the probability that a woman was driving badly at the time of the collision? It is no stretch to say that a Bayesian probability or belief-based ap-

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13. FED. R. EVID. 401(a) (emphasis added).

proach to fact-finding not only invites personal biases into the deliberative process<sup>14</sup> but actually demands it.

This seems like reason enough not to use probabilistic reasoning as a model of legal fact-finding, yet courts and scholars have spent the last fifty years or more struggling to do just that: to describe and apply burdens of persuasion and fact-finding in terms of probabilistic belief that the defendant committed a crime, acted negligently, or breached a contract. Why? Part of the answer is certainly that, even today, we still lack a formal theory of fact-finding, and this is simply the most recent effort to supply that framework. But while we do need a theory of fact-finding, it hardly follows that probabilistic belief concepts are the answer. Even if unbiased, actuarially accurate probabilities could somehow be calculated for the long-run chance that a person would commit a crime, why would it matter? Responsibility for conscious acts and choices is basic to our system of justice. We do not punish a murderer for having some probabilistic tendency toward violence, but for the autonomous act of raising and lowering the knife.

A probabilistic approach to fact-finding invites subjectivity and bias into deliberations in an effort to advance the questionable goal of trying to find historic facts by reference to the fact-finder's belief about the probability of their truth. And problems with the probabilistic approach go deeper still. Decades of efforts to build a Bayesian theory of fact-finding have suffered despairing setbacks: artificial dependence of outcomes on the way that elements of a cause of action are framed, unsettling sensitivity to prior probabilities, and an alarming inability to say what specific facts are found in even simple exercises, to name a few.<sup>15</sup>

These issues are well documented, and while some re-

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14. As explained later in this article, one way to interpret Bayesian probability analysis is as an idealized model of subjective beliefs. *See infra* Section I.A (explaining the close connection between Bayesian posterior probabilities and subjective belief concepts). Other interpretations of beliefs are also possible. *See, e.g.*, Kevin M. Clermont, *Death of Paradox: The Killer Logic Beneath the Standards of Proof*, 88 NOTRE DAME L. REV. 1061 (2013) (describing fuzzy belief functions). Careful distinction between the various possible definitions of belief is not critical to the current argument.

15. *See, e.g.*, Brian Leiter & Ronald J. Allen, *Naturalized Epistemology and the Law of Evidence*, 87 VA. L. REV. 1491, 1503–10 (2001) (discussing several such problems).

searchers still seek a probability theory of fact-finding,<sup>16</sup> many have abandoned the effort. Forfeiting, at least temporarily, any search for a formal theory of fact-finding, these scholars have turned their attention to trying to understand fact-finders' cognitive process at an empirical and descriptive level.<sup>17</sup> Exciting work on relative plausibility,<sup>18</sup> narrative coherency,<sup>19</sup> and story-based models of deliberation<sup>20</sup> advances our understanding of how fact-finders think (iteratively constructing and comparing stories that explain and relate to the evidence), but has relatively little to say about the interpretation of higher burdens of persuasion or the normative scope of idealized fact-finding and boundaries of unfair prejudice, and thus is not a substitute for a formal theory of fact-finding.<sup>21</sup> Nor need we give up the search for a formal theory just yet. The problem with probability-based models of fact-finding is not that mathematics and probability theory are inherently irreconcilable with fact-finding.<sup>22</sup> The problem is that most fact-finding appears not to reflect probabilistic reasoning at all. It appears to reflect reasoning in terms of a related, but different, concept: *likelihood*.

This article presents a likelihood theory of fact-finding. The change from a probabilistic approach is more than semantic. Richard Royall provides a helpful way to conceptualize the difference:<sup>23</sup>

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16. *E.g.*, Edward K. Cheng, *Reconceptualizing the Burden of Proof*, 122 YALE L.J. 1254, 1256–58 (2013).

17. *See generally* Michael S. Pardo, *The Nature and Purpose of Evidence Theory*, 66 VAND. L. REV. 547 (2013) (summarizing recent developments in evidence theory).

18. *E.g.*, Ronald J. Allen, *Factual Ambiguity and a Theory of Evidence*, 88 NW. U. L. REV. 604 (1994).

19. *E.g.*, Dan Simon, *A Third View of the Black Box: Cognitive Coherence in Legal Decision Making*, 71 U. CHI. L. REV. 511 (2004).

20. *E.g.*, Nancy Pennington & Reid Hastie, *The Story Model for Juror Decision Making*, in INSIDE THE JUROR: THE PSYCHOLOGY OF JUROR DECISION MAKING 192 (Reid Hastie ed., 1993).

21. *But see* Michael S. Pardo & Ronald J. Allen, *Judicial Proof and the Best Explanation*, 27 LAW & PHIL. 223, 238–41 (2008) (translating relative-plausibility comparisons to heightened burdens of persuasion).

22. *Cf.* Laurence H. Tribe, *Trial by Mathematics: Precision and Ritual in the Legal Process*, 84 HARV. L. REV. 1329 (1971).

23. *See* Richard Royall, *The Likelihood Paradigm for Statistical Evidence*, in THE NATURE OF SCIENTIFIC EVIDENCE: STATISTICAL, PHILOSOPHICAL, AND EMPIRICAL CONSIDERATIONS 119, 122–23 (Mark L. Taper & Subhash R. Lele eds., 2004) (describing and annotating similar questions).

- Bayesian posterior probability answers the question: “What do I believe, now that I have seen this evidence?”
- Likelihood answers the more basic question: “What does this evidence show?”

Another way to think about the difference is in terms of the cup-choice puzzle. Upon drawing a white marble, most people guess that I have chosen the white cup. They do not base this guess on Bayes’ Theorem, but on the simpler logic that the white cup is more likely to have produced the evidence they’ve seen. This is likelihood reasoning!

I argue that most of legal fact-finding (efforts to decide the truth of historic facts and actions)<sup>24</sup> involves likelihood reasoning. This is not the first paper to suggest the use of likelihood analysis in legal fact-finding,<sup>25</sup> but it *is* the first paper to describe all current fact-finding, and every one of the common burdens of persuasion, in terms of likelihood reasoning alone. To be clear, though, my argument is not that this or any formal model of fact-finding must be imposed on fact-finders in a trial setting. Nor do I argue that this likelihood theory of fact-finding captures every nuance of the legal process; like any formal model, this theory is an abstraction with important limitations.

Instead, what I emphatically do argue is that a reasonable and internally coherent theory of fact-finding should guide our legal practice and procedure. The proposed likelihood theory of fact-finding serves that role. It improves upon efforts to explain fact-finding in terms of probabilities and beliefs, resolving the paradoxes of earlier theories. It simplifies legal analysis, describing every burden of persuasion in terms of the same rule of likelihood reasoning.<sup>26</sup> It harmonizes scholarship, showing how empirical work on the cognitive processes of fact-finders can be formalized into an intuitive and coherent theory. It explains rules of evidence like the exclusion of most character

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24. See H. Richard Uviller, *Evidence of Character to Prove Conduct: Illusion, Illogic, and Injustice in the Courtroom*, 130 U. PA. L. REV. 845, 845 (1982) (“The process of litigation is designed for the reconstruction of an event that occurred in the recent past.”).

25. *E.g.*, Louis Kaplow, *Burden of Proof*, 121 YALE L.J. 738 (2012).

26. *Cf.* Pardo & Allen, *supra* note 21, at 233–41 (describing a fundamentally different reasoning process for fact-finding under the preponderance of the evidence standard versus fact-finding under the beyond a reasonable doubt standard).

reasoning, and evidential mores like judicial hostility to naked statistical evidence. And, perhaps most importantly of all, it offers new insights into the problem of subjective fact-finder belief. Are racial stereotypes, gender stereotypes, or background averages proper bases for legal fact-finding? The likelihood approach answers, “No,” and explains why not.

The arguments here are formal and theoretical, but the implications are immediate and practical. We need to adjust the way we think about and describe legal fact-finding. The formal arguments for a likelihood theory of fact-finding are developed in Part I, which elaborates on the fundamental and philosophical differences between likelihood and probability analysis, and Part II, which develops a complete theory of fact-finding exclusively in terms of likelihood analysis. Readers who would prefer to avoid this more technical exposition can skip to Section II.C for an intuitive summary of the framework. Part III traces the comparative implications of the proposed likelihood theory as a model of idealized fact-finding. Part IV acknowledges the limitations of the likelihood theory as presently developed, and the Article concludes with brief remarks on preliminary implications of this new theory of fact-finding.

To both foreshadow and clarify what is at stake here, consider one last puzzle involving a claim of unjust enrichment tried without a jury. Before any evidence is presented, the judge admits that he’s had business dealings with people from the defendant’s country before, and based on that experience, he is already 80% certain that the plaintiff’s complaint is true. At trial, the plaintiff produces only weak evidence to support her claim, while the defendant puts forth strong evidence that she was not enriched, much less unjustly so. Everyone agrees that the weight of the evidence favors the defendant. Yet because his prior belief was so strong, the judge still personally believes the plaintiff’s claim to be true, and thus finds for the plaintiff. This is a textbook example of probability or belief-based fact-finding. It is the functional equivalent of the racially biased inferences at issue in *Peña-Rodriguez*. But is this really how fact-finding should work? If the process and its outcome strike you as unfair, unjust, and wrong, the likelihood theory of fact-finding helps explain why.

## I. THE LANDSCAPE OF UNCERTAINTY AND PERSUASION

It is impossible to propose a likelihood theory of fact-finding without first saying something about the basic differences between probability and likelihood as concepts of uncertainty. The following does just that by first outlining the concept of probability and the basic mechanics of Bayesian probability analysis, then describing the distinct concept of likelihood and explaining how likelihood analysis differs from probability analysis, and finally turning to a trial context to ask where legal fact-finding and persuasion fit in this landscape of uncertainty.

### A. *Probability: Absolute, Subjective Belief*

Probability is an absolute description of the uncertainty in a system. It translates relative frequencies or beliefs into numbers between zero and one. For example, to say that a random marble drawn from the white cup has a  $5/6$  probability of being white means that if I were to repeat this process an infinite number of times (draw a marble, note its color, replace it, repeat),  $5/6$  of these draws would be white and  $1/6$  would be red. When probability statements depend on other factors, they can be expressed as conditional on these factors. For example, the probability of drawing a white marble depends on which cup I chose:<sup>27</sup>

$$P(\text{draw white marble} \mid \text{white cup chosen}) = 5/6$$

$$P(\text{draw white marble} \mid \text{red cup chosen}) = 1/6$$

Decades of work on fact-finding have focused on trying to swap the order of variables and conditions in this type of probability statement. The fact-finder is assumed to know, at least subjectively, the probability of seeing the evidence given the facts. The objective is then to infer the reverse probability: the probability of the facts given the evidence. But, in general, these terms cannot be simply swapped:

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27. In typical notation, conditioning variables appear to the right of a vertical bar “|” in the probability statement. The notation  $P(A|x)$  is read “probability of  $A$  given  $x$ .”

$$P(\text{white cup chosen} \mid \text{draw white marble}) \neq 5/6$$

Instead, reversing the order of terms requires an appeal to a fundamental law of probability: Bayes' Theorem.<sup>28</sup>

The most common use of Bayes' Theorem in modern fact-finding applications involves the relative probability of two events before and after evidence is observed:<sup>29</sup>

$$\underbrace{\frac{P(A \mid x)}{P(B \mid x)}}_{\text{posterior dist.}} = \underbrace{\frac{P(x \mid A)}{P(x \mid B)}}_{\text{likelihood ratio}} \times \underbrace{\frac{P(A)}{P(B)}}_{\text{prior dist.}}$$

The left-hand term in this equation (the posterior distribution) gives the relative probabilities of two events, *A* and *B*, after evidence *x* has been observed and considered. In the cup-choice puzzle, for example, the evidence is the white marble drawn from the unidentified cup, and the events are my possible choices of cup. The rightmost term in the equation (the prior distribution) describes the relative probabilities of these events before the evidence is observed. And the middle term (the likelihood ratio) conveys the relative consistency of the evidence with each of the possible events. So relative probabilities *after* seeing the evidence equal the relative probabilities *before* seeing the evidence times the likelihood ratio of events on the evidence (more on that later).

The intuitive appeal of Bayes' Theorem in fact-finding applications is hard to miss: just replace *A* and *B* with *guilty* and *innocent*, and you have the probability model of fact-finding that has dominated the evidence literature for the last fifty years.<sup>30</sup> But while much of Bayes' Theorem has been tirelessly explained and re-explained in prior work,<sup>31</sup> one aspect of the model that always seems to get short shrift is how the meaning of the theorem depends on the source of the prior probability

28. See sources cited *supra* note 12 for background on Bayes' Theorem.

29. See, e.g., Sanchirico, *supra* note 11, at 208–10.

30. See John Kaplan, *Decision Theory and the Factfinding Process*, 20 STAN. L. REV. 1065, 1083–91 (1968) (providing what appears to be the first work on this topic); Richard O. Lempert, *Modeling Relevance*, 75 MICH. L. REV. 1021 (1977).

31. See, e.g., Lempert, *supra* note 30, at 1022–25; Richard A. Posner, *An Economic Approach to the Law of Evidence*, 51 STAN. L. REV. 1477, 1486–87 (1999).

distribution.

Things are straightforward when objective prior probabilities are available. As an artificial example, suppose we were to repeat the cup-choice puzzle, but this time I would base my choice on a die roll: I would choose the white cup on a roll of 1–4 and the red cup on a roll of 5–6. This randomization process provides a clear prior distribution for my choice of cup: there is a  $2/3$  prior probability that I will choose the white cup, and a  $1/3$  prior probability that I will choose the red cup. Upon randomly drawing a white marble, you could now use Bayes' Theorem to calculate the relative probability that I would have chosen the white cup:

$$\frac{P(\text{white cup chosen} \mid \text{draw white marble})}{P(\text{red cup chosen} \mid \text{draw white marble})} = \frac{5/6}{1/6} \times \frac{2/3}{1/3} = 10$$

The math is not as important as the interpretation—and in this case, the interpretation is as a long-run ratio of frequencies. If we were to repeat this entire process an infinite number of times, then within the set of repetitions in which you drew a white marble, you should expect me to have chosen the white cup 10 times as often as the red cup. This expectation is not so much a statement about my choice in a given repetition as it is a statement about the *long-run average* of my choices. In short, where objective, long-run, or average prior probabilities are available, the output of Bayes' Theorem is likewise a description of the objective, long-run, or average probabilities of events.

But what happens when clear prior probabilities are not available? In the actual cup-choice puzzle I did not roll a die, but consciously chose a cup. What prior probabilities describe this non-random choice? And, in efforts to apply Bayes' Theorem in a legal context, what are the prior probabilities that a person would breach a contract, neglect a duty, or assault someone? Bayes' Theorem cannot function without prior probabilities, but there is little hope of ever finding objective, long-run, or average probabilities of these events.<sup>32</sup>

When clear, objective prior probabilities are unavailable,

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32. See Ronald J. Allen & Alex Stein, *Evidence, Probability, and the Burden of Proof*, 55 ARIZ. L. REV. 557, 566–67 (2013) (“[F]requentist probability is of no use . . . . Courts have no information about the relative frequencies of relevant events.”).

Bayesian analysis falls back on subjective prior probabilities. That is, probabilities that reflect the personal assumptions or individual beliefs of the user.<sup>33</sup> And when this happens, the interpretation of Bayes' Theorem is no longer a description of long-run or average probability; it is more like a normative claim about beliefs. Combining the inputs of a person's initial beliefs and the observed evidence, Bayes' Theorem describes what a fully rational person with these prior beliefs should now believe upon seeing the evidence.

In some settings, this incorporation of subjective prior beliefs into the analytic process makes sense. When deciding whether to accept an offer or continue to haggle, for example, it only makes sense to start from one's own belief about what a potential compromise might be.<sup>34</sup> But in other settings, the intensely personal nature of Bayesian reasoning can be disquieting. If a person is a racist, for example, then Bayes' Theorem describes what a fully rational racist should believe after seeing some item of evidence. If a person is a misogynist, then Bayes' Theorem describes what a fully rational misogynist should believe. And since beliefs may vary from person to person even in the absence of any overt prejudice or bias, two people could see the same evidence and completely agree on its probative weight, yet form different beliefs about the facts as a result of their different prior beliefs.<sup>35</sup>

### B. *Likelihood: Relative Weight of the Evidence*

Though the terms *probability* and *likelihood* are synonyms in informal English,<sup>36</sup> modern statistics draws a distinction be-

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33. See A.W.F. EDWARDS, *LIKELIHOOD: EXPANDED EDITION* 51 (1992) (“[I]n order to apply Bayes' Theorem to hypotheses not generated by a chance set-up, prior probabilities, for which there is no frequency justification, will have to be invented.”).

34. As a description of how a person might structure their thinking or order their preferences, Bayes' Theorem is an attractive model. See Ward Edwards, Harold Lindman, & Leonard J. Savage, *Bayesian Statistical Inference for Psychological Research*, 70 *PSYCHOL. REV.* 193, 195 (1963) (“The Bayesian approach is a common sense approach. It is simply a set of techniques for orderly expression and revision of your opinions with due regard for internal consistency among their various aspects and for the data.”).

35. See, e.g., RICHARD M. ROYALL, *STATISTICAL EVIDENCE: A LIKELIHOOD PARADIGM* 169–70 (1997) (providing a simple illustration of this point).

36. E.g., *Probability*, BLACK'S LAW DICTIONARY (10th ed. 2014) (“Something that is likely . . . . The degree to which something is likely to occur . . . . The

tween these ideas. If Bayes' Theorem is concerned with the probability of an event given observed evidence,  $P(A|x)$ , then likelihood is concerned with the probability of observing the evidence given a hypothesized event,  $P(x|A)$ . Where Bayesian posterior probabilities often depend critically on prior beliefs, likelihood is a belief-agnostic statement about the consistency of evidence with contrasting hypotheses about the world.<sup>37</sup>

In formal notation, a likelihood function is represented as follows:

$$L(\theta; x) = c \times P(x|\theta)$$

where  $c > 0$  can be any arbitrary constant.<sup>38</sup> The argument of interest in the likelihood function,  $\theta$ , is a variable that takes on all the possible hypotheses of interest. For example, in the cup-choice puzzle,  $\theta$  has two possible values: (1) "white cup chosen," and (2) "red cup chosen." The conditioning variable,  $x$ , is the evidence that has been observed. In the cup-choice puzzle, this is the observation of a white marble drawn at random from the unidentified cup. The likelihood function is proportional to the probability of seeing the observed evidence,  $x$ , under different hypotheses about the world,  $\theta$ , and is thus a tool for comparing the consistency of different hypotheses with the evidence.

Likelihoods are related to probability statements by definition. But likelihoods are not probabilities.<sup>39</sup> At a deep level, probabilities and likelihoods behave differently:

- Probabilities have individual meaning (a 1/2 probability of an event is an absolute statement of frequency or belief in

quality, state, or condition of being more likely to happen or to have happened than not . . ."). In fact, the inventor of the concept of likelihood, Ronald A. Fisher, introduced it in an effort to clarify linguistic confusion among statisticians. R. A. Fisher, *On the Mathematical Foundations of Theoretical Statistics*, 222 PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY OF LONDON A 309, 309–11 (1921).

37. See ROYALL, *supra* note 35, at 170–73 (advancing and explaining this interpretation of likelihoods and likelihood analysis).

38. The point of the constant,  $c > 0$ , is to act as a reminder that likelihood has no meaning on its own. A likelihood of  $L(A) = 8$  says nothing about the probability of  $A$ ; it only reflects that  $A$  is twice as likely as  $B$  if  $L(B) = 4$ , since  $L(A)/L(B) = 2$ . This ratio is the same for any scaling constant,  $c > 0$ . See *id.* at 24 (providing further explanation of the likelihood function and this property).

39. See, e.g., YUDI PAWITAN, IN ALL LIKELIHOOD: STATISTICAL MODELLING AND INFERENCE USING LIKELIHOOD 29 (2013) ("[T]he fundamental difference is that *the likelihood does not obey probability laws*. So probability and likelihood are different concepts available to deal with different levels of uncertainty.").

that proposition); likelihoods have no individual meaning, and can only be interpreted in comparison to other likelihoods.<sup>40</sup>

- Probabilities add to one over all possible values of the random variable; likelihoods need not add to one (or even a finite number) over all possible values of their variable of interest.<sup>41</sup>
- Probabilities allow simple hypotheses, like  $P(A)$ , to be compared to composite hypotheses, like  $P(B \text{ or } C)$ ; likelihoods can generally only be used to compare two simple hypotheses at a time.<sup>42</sup>

It may sound like likelihoods lack many of the attractive properties of probabilities—and they do. Compared to probability, likelihood is a weaker (less descriptive) concept of uncertainty.<sup>43</sup> But likelihoods have their own attractive properties arising from the special features of likelihood ratios. Chief among these is the *Law of Likelihood*, the takeaway of a set of related arguments that for a given probability model, seeing evidence  $x$  favors hypothesis  $A$  over hypothesis  $B$  if and only if the likelihood of  $A$  exceeds the likelihood of  $B$ :

$$LR = \frac{L(A; x)}{L(B; x)} > 1$$

The Law further provides that the total strength of evidential support is reflected in the size of this likelihood ratio.<sup>44</sup>

40. *E.g., id.* at 226 (“[A] likelihood ratio compares the relative merits of two hypotheses in light of the data; it does not provide an absolute support for or against a particular hypothesis on its own.”); ROYALL, *supra* note 35, at 24 (making a similar observation); *see also supra* note 38.

41. *E.g.,* IAN HACKING, LOGIC OF STATISTICAL INFERENCE 50 (1965) (noting that likelihoods do not obey the Kolmogoroff axioms); Fisher, *supra* note 36, at 327 (similar).

42. *E.g.,* ROYALL, *supra* note 35, at 16–20 (explaining and detailing this property).

43. *See, e.g.,* PAWITAN, *supra* note 39, at 28 (noting that Fisher considered likelihood weaker than probability, appropriate to “analyze . . . and communicate statistical evidence of types too weak to supply true probability statements”) (citing RONALD A. FISHER, STATISTICAL METHODS AND SCIENTIFIC INFERENCE 75 (3d ed. 1973)).

44. This description of the *Law of Likelihood* is based on Ian Hacking’s formulation. HACKING, *supra* note 41, at 48–66; *see also* EDWARDS, *supra* note 33, at 28–31 (similar); ROYALL, *supra* note 35, at 1–3 (similar). For a formal argument of the related likelihood principle, *see* Allan Birnbaum, *On the*

In the cup-choice puzzle, for example, drawing a white marble from the unidentified cup is evidence in favor of the hypothesis that I chose the white cup over the alternative that I chose the red cup:

$$\frac{L(\text{white cup chosen; white marble drawn})}{L(\text{red cup chosen; white marble drawn})} = \frac{c \times 5/6}{c \times 1/6} = 5$$

A likelihood ratio of 5 means that the available evidence is 5 times more consistent with my having chosen the white cup than the red cup. This ratio would have the same interpretation in a medical context, a physical science context, or a legal fact-finding context.<sup>45</sup> Likelihood ratios provide a general language for describing the relative consistency of competing hypotheses with observed evidence.

But what a likelihood ratio of 5 emphatically does not say is that the *probability* of *A* is 5 times the probability of *B*. The hypotheses, *A* and *B*, may concern historic acts or fixed constants for which one need not assume anything about prior probabilities. Likelihood analysis of the cup-choice puzzle thus imposes no restriction at all on my choice of cup. The analysis would work just as well even if *no* prior probabilities could describe my choice process.<sup>46</sup> In this way, likelihood reasoning differs in a fundamental way from probability reasoning. Likelihood analysis has no need for either objective or subjective prior probabilities, and the output of likelihood analysis is likewise neither a statement of long-run probability nor belief. Likelihood concerns and describes the comparative consistency of different hypotheses with the evidence. On the basis of observed evidence—and evidence alone—likelihood “supplies a natural order of preference among the possibilities under consideration.”<sup>47</sup>

The connection to fact-finding is again hard to miss: likelihood provides an independent language for describing fact-finding. If certain evidence is more likely to arise under one set of

*Foundations of Statistical Inference*, 57 J. AM. STAT. ASS'N 269 (1962).

45. See ROYALL, *supra* note 35, at 27.

46. See EDWARDS, *supra* note 33, at 55–65 (discussing the impossibility of representing ignorance by a prior probability distribution); ROYALL, *supra* note 35, at 173 (same).

47. RONALD A. FISHER, *STATISTICAL METHODS AND SCIENTIFIC INFERENCE* 68 (1956) (quoted in EDWARDS, *supra* note 33, at 27).

facts than another, then its observation supports the more likely factual hypothesis. The size of the likelihood ratio reflects the strength of this support. In fact, the size of the likelihood ratio arguably summarizes all of the information contained in the entire body of observed evidence.<sup>48</sup>

### C. *Persuasion: Fact-Finding Under Uncertainty*

Like both probability and likelihood, persuasion is a concept arising from uncertainty. If fact-finders could divine the facts of a case without error or doubt, then persuasion would never enter the picture. The facts would simply be compared to the cause of action to perfectly resolve every dispute. The world is, of course, not so perfect. Material facts often remain uncertain at the close of evidence,<sup>49</sup> and so burdens of persuasion are used to categorize evidence that is good enough to legally establish still uncertain facts, claims, and defenses.

So, if probability describes belief, and likelihood describes evidential support, which is the measure of persuasion? It would be nice if courts or lawmakers had ever consciously adopted one measure or the other. But they haven't. And common articulations of every burden of persuasion are today a muddled mess of imprecise notions of probability, likelihood, certainty, doubt, and the weight of evidence.

For example, in most civil actions the burden of persuasion is proof by preponderance of the evidence. This is often explained as requiring the fact-finder to "be persuaded by the evidence that the claim [or affirmative defense] is more probably true than not true."<sup>50</sup> But it is also explained as requiring "any underlying material fact [to be] more likely [true] than

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48. See, e.g., EDWARDS, *supra* note 33, at 30 ("Within the framework of a statistical model, *all* the information which the data provide concerning the relative merits of two hypotheses is contained in the likelihood ratio of those hypotheses on the data.").

49. See McBaine, *supra* note 10, at 246 ("[C]ertainty as to what has happened cannot be ascertained from the testimony of witnesses or other evidence of acts. The frailty of man is such that certainty in the field of fact finding is impossible.").

50. NINTH CIRCUIT MANUAL OF MODEL CIVIL JURY INSTRUCTIONS § 1.6 (2017) (brackets in original), [http://www3.ce9.uscourts.gov/jury-instructions/sites/default/files/WPD/Civil\\_Instructions\\_2018\\_6.pdf](http://www3.ce9.uscourts.gov/jury-instructions/sites/default/files/WPD/Civil_Instructions_2018_6.pdf) [<https://perma.cc/29DW-ZDA6>]; see also SEVENTH CIRCUIT FEDERAL CIVIL JURY INSTRUCTIONS, *supra* note 8, § 1.27 ("[Y]ou must be persuaded that it is more probably true than not true."). Cf. THIRD CIRCUIT MODEL CIVIL JURY INSTRUCTIONS, *supra* note 8, § 1.10 ("[W]hat [the plaintiff] claims is more likely so than not so.").

not.”<sup>51</sup> And as requiring that “the scales tip, however slightly, in favor of the party with [the] burden [of persuasion].”<sup>52</sup>

In most criminal actions, due process demands a more stringent standard: proof beyond a reasonable doubt.<sup>53</sup> Explanations of this burden rarely use either probability or likelihood language. Instead, proof beyond a reasonable doubt is described as something like “proof that leaves you firmly convinced [that] the defendant is guilty.”<sup>54</sup> Or in the negative, it is said to be lack of a doubt that “would cause a prudent man to hesitate in taking action upon an important matter,” as opposed to only “arbitrary” or “speculative” doubt.<sup>55</sup> Some jurisdictions refuse to attempt any definition at all.<sup>56</sup>

In between these extremes, some civil claims and defenses are held to intermediate standards like proof by clear and convincing evidence. These intermediate burdens are neither as standardized as the main two, nor as well understood.<sup>57</sup> And that is failing to meet a low bar, since confusion about even the main two burdens of persuasion has embarrassed the legal system for generations.<sup>58</sup>

Centuries of efforts to clarify the burdens of persuasion by

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51. *Aguilar v. Atl. Richfield Co.*, 24 P.3d 493, 507 (Cal. 2001); *see also* *Guglielmino v. McKee Foods Corp.*, 506 F.3d 696, 698 (9th Cir. 2007) (defining the preponderance standard as that in which a claim is shown “more likely than not”).

52. *Ostrowski v. Atl. Mut. Ins. Cos.*, 968 F.2d 171, 187 (2d Cir. 1992).

53. *See In re Winship*, 397 U.S. 358, 364 (1970) (holding “that the Due Process Clause protects the accused against conviction except upon proof beyond a reasonable doubt of every fact necessary to constitute the crime with which he is charged”).

54. NINTH CIRCUIT MANUAL OF MODEL CRIMINAL JURY INSTRUCTIONS § 3.5 (2010), [http://www3.ce9.uscourts.gov/jury-instructions/sites/default/files/WPD/Criminal\\_Instructions\\_2017\\_9.pdf](http://www3.ce9.uscourts.gov/jury-instructions/sites/default/files/WPD/Criminal_Instructions_2017_9.pdf) [<https://perma.cc/UP2M-ZXX4>].

55. *McBaine*, *supra* note 10, at 257 (summarizing these and other expressions).

56. *See, e.g.,* *United States v. Glass*, 846 F.2d 386, 387 (7th Cir. 1988) (“This case illustrates all too well that attempts to explain the term ‘reasonable doubt’ do not usually result in making it any clearer to the minds of the jury. And that is precisely why this circuit’s criminal jury instructions forbid them. ‘Reasonable doubt’ must speak for itself.” (internal markup and citations omitted)).

57. *See* 2 MCCORMICK ON EVIDENCE § 340 (Kenneth S. Broun ed., 7th ed. 2013) (citing additional examples such as “clear, convincing and satisfactory,” “clear, cogent, and convincing,” and “clear, unequivocal, satisfactory and convincing [evidence]” of these standards, and commenting that “[n]o high degree of precision can be attained by these groups of adjectives”).

58. *See McBaine*, *supra* note 10, at 246 (arguing, in 1944, that “[w]e should not . . . have uncertainty, conflict and confusion in our legal system as to what are [the three common burdens of] persuasion and belief or how they should be adequately expressed in instructions for a jury”).

linguistic refinement having failed<sup>59</sup>—the past fifty years have seen courts and scholars turn to more formal probability concepts in their efforts to say what each of these standards means. Edward Cheng recently summarized the probability thresholds that are now quite often used in attempts to explain the most common burdens of persuasion: “As every first-year law student knows, the civil preponderance-of-the-evidence standard requires that a plaintiff establish the probability of her claim to greater than 0.5. By comparison, the criminal [reasonable-doubt] standard is akin to a probability greater than 0.9 or 0.95.”<sup>60</sup> These thresholds are expressed in terms of the probability of a single proposition. As ratios of probabilities (the probability of guilt versus innocence, for example) they would be more like 1.0 for preponderance and 9.0–19.0 for reasonable doubt, but the idea is the same.<sup>61</sup> Intermediate standards are less often targeted for quantification and presumably fall somewhere between these extremes.

Many scholars harbor serious doubts about the Bayesian approach to fact-finding,<sup>62</sup> but with no alternative theory to take its place, the probability approach has long dominated academic research.<sup>63</sup> Courts are slowly adopting this approach too. While most continue to resist quantifying the reasonable doubt standard,<sup>64</sup> some now explicitly endorse a 50% probability threshold when explaining preponderance of the evidence to jurors.<sup>65</sup> And surveys of judges show rough consensus that pre-

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59. MCCORMICK ON EVIDENCE, *supra* note 57, § 339 (commenting on effort wasted arguing over linguistic metaphysics and “word-magic” in the precise way that burdens of persuasion are explained to fact-finders).

60. Cheng, *supra* note 16, at 1256; *see also* Allen, *supra* note 18, at 604 (describing the same probability ranges and commenting that until the 1990s, “[n]ot only was this view conventional, it was virtually uncontroversial”).

61. For example, a 0.9 probability of guilt, implying a  $1.0 - 0.9 = 0.1$  probability of innocence, corresponds to a  $0.9/0.1 = 9.0$  probability ratio of guilt to innocence.

62. *See generally* Allen, *supra* note 18 (highlighting the conjunction paradox and empirical research on how fact-finder decision-making differs from the predictions of a Bayesian probability theory of fact-finding); Allen & Stein, *supra* note 32 (discussing the conjunction paradox and specific deficiencies in recent fact-finding models).

63. *See, e.g.*, Gary L. Wells, *Naked Statistical Evidence of Liability: Is Subjective Probability Enough?*, 62 J. PERSONALITY & SOC. PSYCHOL. 739, 739 (1992) (noting that “the probability-threshold model” is “the dominant decision model put forth in the [literature]” and citing decades of research on this model).

64. *E.g.*, *Commonwealth v. Sullivan*, 482 N.E.2d 1198, 1199–1200 (Mass. App. Ct. 1985) (finding error in trial court’s illustrative quantification of reasonable doubt standard in probability terms in response to jury request for clarification).

65. *E.g.*, *Brown v. Bowen*, 847 F.2d 342, 345–46 (7th Cir. 1988) (“All burdens

ponderance of the evidence requires greater than a 50–60% probability of a fact’s truth,<sup>66</sup> while reasonable doubt requires something higher, like a 75–90% probability of truth.<sup>67</sup>

Likelihood reasoning is not entirely missing from the conversation, but it is rarely treated as more than a stepping-stone on the path to the posterior probabilities of interest, and never considered an independent concept of uncertainty for the fact-finding process. The closest the literature has come to a likelihood theory of fact-finding has been in recent papers by Louis Kaplow and Edward Cheng. In 2012, Kaplow proposed a new system of fact-finding oriented around “evidence thresholds” derived from assumptions about the social objectives of the justice system. Kaplow’s novel system of fact-finding would use a form of likelihood reasoning, but his reading of current fact-finding and burdens of persuasion is consistent with the usual Bayesian probability approach.<sup>68</sup> In 2013, Cheng proposed to model fact-finding in terms of posterior probability ratios, but subject to ad hoc assumptions that happened to make these Bayesian probabilities behave like likelihoods in many respects.<sup>69</sup> And in 2014, Kaplow again suggested that likelihood ratios could be used to model various legal decision-making

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of persuasion deal with probabilities. The preponderance standard is a more-likely-than-not rule, under which the trier of fact rules for the plaintiff if it thinks the chance greater than 0.5 that the plaintiff is in the right. The reasonable doubt standard is much higher, perhaps 0.9 or better. The clear and convincing standard is somewhere in between.”).

66. See, e.g., *United States v. Fatico*, 458 F. Supp. 388, 409–10 (E.D.N.Y. 1978) (citing and surveying evidence from two such surveys).

67. See, e.g., Posner, *supra* note 31, at 1506 (surveying studies and noting that “[j]udges, when asked to express proof beyond a reasonable doubt as a probability of guilt, generally pick a number between .75 and .90”). See generally Reid Hastie, *Algebraic Models of Juror Decision Processes*, in *INSIDE THE JUROR: THE PSYCHOLOGY OF JUROR DECISION MAKING* 192 (Reid Hastie ed., 1993) (summarizing many studies eliciting probability thresholds for the preponderance and reasonable doubt standards).

68. Kaplow, *supra* note 25, at 748 (distinguishing current law, which “takes behavior as given . . . and asks, in light of that behavior, what is the likelihood of [harmful or benign] acts” from the proposed “welfare-based, optimal threshold” approach in which the central question is “how behavior . . . will change as a function of a change in the evidence threshold”). Care is needed in interpretation, however, as Kaplow does not always distinguish probability from likelihood in his terminology. See, e.g., *id.* at 758–59; *id.* at 748 n.19.

69. Cheng, *supra* note 16, at 1263–65 (assuming probabilities of alternative fact-combinations cannot be aggregated in fact-finding); *id.* at 1267–68 & nn.24–25 (assuming the ratio of prior probabilities must always equal one in a legal setting).

problems, but again interpreted current burdens of persuasion in terms equivalent to the traditional Bayesian approach.<sup>70</sup>

Suffice it to say, while recent work is skirting the edges of a likelihood theory of fact-finding, the critical step of deriving a pure theory of fact-finding from likelihood reasoning alone has yet to be undertaken. The next section does so.

## II. A LIKELIHOOD THEORY OF FACT-FINDING

It is hard to think clearly about fact-finding without the foundation of a framework and lexicon for how facts, evidence, and causes of action interact in the justice system.<sup>71</sup> This is not to say that any of these terms are the least bit alien, only that they are rarely defined with precision in legal research. As this section shows, even a tentative effort to standardize terms quickly leads to a likelihood theory of fact-finding.

### A. *The General Fact-Finding Framework*

Reduced to core concepts, legal fact-finders are asked to compare uncertain facts to the elements of a cause of action in reaching a verdict. They do so by reference to evidence put forth by litigants, and subject to whatever burden of persuasion represents sufficient evidence to prove the facts in a given context. None of this is new ground, but it is still instructive to take the uncommon step of considering each component of this framework in turn.

*Facts* are the actions, omissions, intents, and beliefs of the parties that are material to a claim for legal relief. These facts could include some random elements (like accidental injury resulting from the defendant's negligence) but will more often consist of historic acts and choices (like the defendant's conscious decision not to undertake certain safety measures). Mixed questions of fact and law (whether failure to undertake

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70. Louis Kaplow, *Likelihood Ratio Tests and Legal Decision Rules*, 16 AM. L. ECON. REV. 1, 35 (2014) (“[Preponderance of the evidence] can be equivalently stated in terms of the posterior probabilities . . . or in terms of the likelihood ratio . . .”).

71. Oddly, research on fact-finding often skips this important and seemingly obvious step. *But cf.* David H. Kaye, *Do We Need a Calculus of Weight to Understand Proof Beyond a Reasonable Doubt?*, 66 B.U. L. REV. 657, 659–61 (1986) (distinguishing between facts, evidence, and narrative stories in a manner similar to that presented here).

these safety measures breached a duty of care) are themselves simply functions of more basic facts (what measures could have been taken; what their efficacy would have been), and so are no different from standard facts at a theoretical level. The universe and granularity of possible facts is an empirical question of how exactly a case is presented to and perceived by the fact-finder.<sup>72</sup>

A *cause of action* is a set of facts sufficient to warrant legal relief. If  $F$  denotes the universe of all combinations of material facts that could plausibly be true, then let  $C \subset F$  denote the subset of that universe in which the combination of facts makes out a cause of action.<sup>73</sup> In a negligence action, for example,  $C$  would be all combinations of facts that satisfy the elements of duty, breach of duty, causation, and damages; in a criminal action, it would be all combinations of facts that fit the elements of the crime charged. If a given combination of facts falls within the cause-of-action set,  $f \in C$ , then it justifies legal relief. The complement of the cause-of-action set,  $C^c$ , is the set of all possible combinations of facts in  $F$  that are not in  $C$ . Any combination of facts falling in this no-remedy set,  $f \in C^c$ , fails to justify relief.

*Evidence* is what the fact-finder sees and hears at trial.<sup>74</sup> It is sometimes forgotten that the evidence is not the facts. It is not usually even direct proof of the facts. At trial, the fact-finder hears testimony and sees documentary evidence that bears on what the facts might be.<sup>75</sup> Some of this evidence is di-

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72. See Pardo & Allen, *supra* note 21, at 233–38 (providing a descriptive account of the types of factors that may help influence fact-finders’ perception of the choice-space of material facts, including the legal elements of the cause of action and the focalizing points of contrast between the parties’ versions of events).

73. The set notation in this paragraph is basic, and describes relationships succinctly. The notation  $x \in A$  means “ $x$  is an element of the set  $A$ .” For example, if  $A$  is a set with two elements,  $A = \{1, 4\}$ , then  $4 \in A$  but  $3 \notin A$ . The notation  $A \subset B$  means “ $A$  is a subset of  $B$ .” For example, if  $A = \{1, 4\}$  and  $B = \{1, 4, 5\}$  then  $A \subset B$  because  $B$  contains both 1 and 4, but  $B \not\subset A$  because  $A$  is missing 5.

74. This definition abstracts from practical complexities—such as the interpretation of evidence admitted subject to limiting instruction—that are not themselves relevant to the argument of the paper. See, e.g., FED. R. EVID. 105 (describing the practice of restricting an item of evidence “to its proper scope,” and instructing jurors that their use of the item of evidence cannot exceed this scope).

75. Cf. 1 CHRISTOPHER MUELLER & LAIRD KIRKPATRICK, FEDERAL EVIDENCE 544–45 (4th ed. 2013) (“Sometimes circumstantial evidence bears immediately on an element of a claim or defense, but the proof is fragmented and consists of many facts. Alone, each has little or no probative force, but the whole . . . may almost

rect proof of the facts. Some of it is evidence explaining why other evidence has or hasn't been shown: impeachment of a witness by bias or prior inconsistent statement,<sup>76</sup> presence or absence of records of a regularly conducted activity,<sup>77</sup> etc. Some of it is evidence presented to build a broader narrative or story.<sup>78</sup> From this body of disparate and usually conflicting evidence, the fact-finder is asked to draw inferences about the facts of the case.

Unlike the facts, it will generally make sense to think of the body of evidence as largely generated by a random process. To see why, consider even a simple negligence action in which the motivating injury was seen by many bystanders. As a practical matter, neither party can force a given bystander to take the stand as a friendly witness, and cooperation is largely luck-of-the-draw. Even if a witness is willing to cooperate, the quality of her testimony may be limited by memory and communication skills. And even this potential testimony will only reach the fact-finder if the lawyers remember to introduce it and the rules of evidence allow its admission.<sup>79</sup> The same goes for documentary evidence. Documents may not be retained long enough to be discovered, may be retained but still not discovered, may be discovered but not reviewed, may be reviewed but not properly interpreted, may be properly interpreted but impossible to authenticate,<sup>80</sup> may be possible to authenticate but difficult to read or present, and so on. In sum, the particular body of evidence that ultimately gets on the record is the result of a long and complicated process over which neither party has great control—a process with a large random component.<sup>81</sup>

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impel a conclusion that some or all elements of a claim or defense have been shown. In such cases the proof can properly be viewed as a whole . . .").

76. *E.g.*, *United States v. Abel*, 469 U.S. 45, 52 (1984) (explaining bias); FED. R. EVID. 607 (attacking credibility); FED. R. EVID. 613 (prior inconsistent statement).

77. *E.g.*, FED. R. EVID. 803(6)–(7) (presence and absence of certain records).

78. *E.g.*, *State v. Villavicencio*, 388 P.2d 245, 246 (Ariz. 1964) (“[The] principle that the complete story of the crime may be shown even though it reveals other crimes has often been termed ‘res gestae’ . . . we choose to refer to this as the ‘complete story’ principle.”); *see also* Kaye, *supra* note 71, at 662–65 (discussing the role of stories and “gaps” in evidence in a probability theory of fact-finding).

79. *E.g.*, FED. R. EVID. 801–807 (hearsay); FED. R. EVID. 602 (personal knowledge); FED. R. EVID. 403 (balancing probative value against risk of undue prejudice, etc.).

80. *E.g.*, FED. R. EVID. 901.

81. The claim here is not that evidence availability is *entirely* outside the control of the litigants. Rather, the claim is that evidence availability is heavily

This randomness does not mean that the evidence is unrelated to the facts. To the contrary, if the plaintiff really did suffer an injury as the result of the defendant's negligence, it seems probable that at least one of the bystanders would be willing to take the stand to testify to that effect. And, similarly, the more negligent the defendant's acts, the less probable it is that she would be able to find a bystander whose imperfect recollection painted her as prudent. Put formally, the probability distribution generating the evidence depends on the underlying facts.<sup>82</sup> So if  $E$  is the universe of all evidence that could possibly make the record, then the probability of observing a given combination of evidence,  $e \in E$ , will usually vary with the hypothesized facts:  $P(e | f)$ .

For purposes of laying out the likelihood theory itself, the existence of these conditional probabilities,  $P(e | f)$ , can be taken as primitive: something the fact-finder knows or can assess. This assumption is basic to a Bayesian probability approach to fact-finding as well,<sup>83</sup> and is actually more restrictive in that context than it is here. Probability reasoning usually requires the fact-finder to be able to assess the absolute probability of observing a particular body of evidence, whereas likelihood reasoning only requires that the fact-finder be able to assess the comparative probability of observing the evidence under different factual hypotheses.

In trial fact-finding, the practical analogy to this type of likelihood reasoning is described by Ron Allen and many others in their work on the cognitive process of fact-finders.<sup>84</sup> To over-

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influenced by factors that *are* outside the control of the parties.

82. See, e.g., Allen & Stein, *supra* note 32, at 577 ("Virtually always, therefore, this evidence will have some causal connection to the story's truth. To put it differently, this evidence would not have come into existence the way it did had the story been false rather than true."); Lempert, *supra* note 30, at 1052 ("Upon hearing testimony, jurors must compare the probability that the testimony *would be given* if the defendant were guilty with the probability that the testimony *would be given* if the defendant were innocent."); see also Uviller, *supra* note 24, at 846 ("As we assemble our evidence, we usually adopt the footprint theory of how to show who passed by. According to this theory, the past is preserved in the present by altered surfaces of matter and mind.").

83. This assumption has undergirded the Bayesian analysis of fact-finding since the earliest works on this topic. E.g., Kaplan, *supra* note 30, at 1085 ("[T]he [fact-finder] will . . . determine as best he can . . . the probability that the piece of evidence would have occurred under the hypothesis of guilt to the probability that it would have occurred under the hypothesis of innocence.").

84. See, e.g., sources cited *supra* notes 18–20; see also Allen & Stein, *supra* note 32, at 567–71 (discussing and citing literature related to the relative

simplify this important body of work, fact-finders appear to assess the merits of competing factual narratives by comparing the epistemic credentials of different factual theories given the evidence. The type of factors considered in this process include the coherence, consilience, and causal specificity of stories about the facts, and other assessments of the comparative probability of observing the evidence under different factual hypotheses.<sup>85</sup> Put another way, empirical research suggests that fact-finders interpret evidence in relation to factual narratives in a way that is difficult to distinguish from likelihood reasoning.

### B. *The Likelihood Approach to Fact-Finding*

A theory of legal fact-finding based on likelihood reasoning explains every burden of persuasion in use today. All that this theory depends upon is the context provided by the foregoing fact-finding framework and the special properties of likelihood ratios. The theory closely fits much of the language, practice, and intuition of the current legal fact-finding process.

The easiest way to explain this theory is to start with how it defines the ultimate fact-finding inquiry. For simplicity of exposition, suppose that the plaintiff bears the burden of proof on all material fact questions in the case at hand. Every burden of persuasion in use today can be reduced to the exact same rule of likelihood reasoning: find for the plaintiff if and only if

$$LR = \frac{\sup_{f \in C} L(f; e)}{\sup_{f \in C^c} L(f; e)} > k$$

where  $k \geq 1$  is a threshold value determined by the applicable burden of persuasion. Let me explain.

Broken down, this test has two parts: to the left of the inequality is a likelihood ratio; to the right, a threshold value. The

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plausibility model).

85. See Allen & Stein, *supra* note 32, at 577 (“[E]vidence that allows the winning story to win . . . does not come into existence by accident. This evidence must satisfy a demanding set of epistemic criteria [such as coherence, causal specification, evidential support, and other criteria associated with natural reasoning.]”); see also sources cited *supra* note 82 (suggesting a probabilistic link between random evidence and the facts).

reliance on likelihood ratios shouldn't be surprising. The Law of Likelihood provides that the observed value of a random variable (evidence) is support for one hypothesis (factual theory) over another (opposing theory) *if and only if* the likelihood of the first hypothesis is greater than the likelihood of the second hypothesis.<sup>86</sup>

The point of the threshold value is also explained by the Law of Likelihood. Recall that the magnitude of a likelihood ratio conveys just how strongly the evidence favors the top hypothesis over the bottom hypothesis.<sup>87</sup> Accepting the legal truth of the top hypothesis only when its likelihood is more than  $k$  times greater than that of the bottom hypothesis amounts to building a strength-of-evidence requirement into fact-finding. As the value of  $k \geq 1$  rises, the evidence must more strongly favor the top hypothesis over the bottom hypothesis before the top hypothesis will be taken as legally true.

The *sup* (supremum) terms in both the numerator and denominator of the likelihood ratio instruct the fact-finder to select and compare only the most likely combinations of facts in two mutually exclusive and exhaustive subsets of the universe of possible facts. The top term is the likelihood of the most plausible combination of facts in the cause-of-action set: the likelihood of the most likely pro-plaintiff factual theory. The bottom term is the same, but for the no-remedy set: the likelihood of the most likely pro-defendant factual theory. Put another way, this likelihood ratio compares the single most plausible factual theory favoring each side of the case—pitting evidential support for the single strongest factual story for the plaintiff against evidential support for the single strongest factual story for the defendant.

Basing this ultimate comparison on only these two most likely factual stories aligns with the typical use of likelihood ratios in statistics,<sup>88</sup> and makes intuitive sense in the trial context, where both sides are given an opportunity to explain how

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86. See sources cited *supra* note 44 and accompanying text (discussing the Law of Likelihood).

87. See sources cited *supra* notes 44–48 and accompanying text (discussing this property).

88. The comparison of suprema of likelihoods within different categories of hypotheses is how likelihood ratio tests are typically conducted in statistics. See generally GEORGE CASELLA & ROGER L. BERGER, STATISTICAL INFERENCE 373–79, 385–91 (2d ed. 2002) (discussing the construction of statistical likelihood ratio tests).

the evidence supports their respective factual theories. We might suppose that the top and bottom terms of the likelihood ratio will often reflect the factual stories advanced by the respective parties, but nothing in this model or the rules of evidence requires it. Suppose, for example, that the fact-finder considers some other pro-plaintiff theory more plausible than the theory advanced by the plaintiff. In this case, the fact-finder will simply ignore the plaintiff's suggested theory, and that other, more plausible factual theory will occupy the numerator of the likelihood ratio.<sup>89</sup>

It may seem surprising that the final analysis is limited to only these two most likely factual theories, but on both technical and intuitive grounds that limitation makes a great deal of sense. First, to be clear, this does not prevent either party from arguing in the alternative. It simply means that the single most plausible argument among the alternative arguments is the one that will be considered in the final comparison. The alternatives will not be aggregated in the analysis. Second, the non-aggregation of alternative hypotheses is not some artificial restriction imposed on the framework, but a substantive implication of the fact that prior probabilities do not factor into likelihood analysis. Precisely because it eschews reliance on prior probabilities, likelihood reasoning does not afford any coherent way to understand what it means to aggregate the likelihoods of alternative hypotheses in most cases.<sup>90</sup> Third, the ultimate choice between two fully specified factual theories is an intuitively desirable property of *any* theory of legal fact-finding. It means that the end product of legal fact-finding will always be a specific finding of facts—something that an approach based on the aggregation of alternative factual hypotheses could never promise.

Finally, the likelihood ratio comparing the plausibility of these two factual theories is itself compared to a threshold value derived from the burden of persuasion. As explained in

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89. Allen & Stein, *supra* note 32, at 568 (“[Fact-finders] consider the parties’ competing stories and decide which is superior; in some cases, they construct their own account of the events in light of the parties’ evidence and arguments.”).

90. See sources cited *supra* notes 39–42 and accompanying text. Intuitively, without the basis of a prior probability distribution, it is impossible to say how much weight to assign each likelihood in an attempted aggregation. The lack of prior distribution does not mean that each possibility has equal prior probability, as would be needed, for example, to simply add up the various raw likelihood of alternative hypotheses.

the following subsections, that threshold value is  $k = 1$  for preponderance of the evidence, something like  $1 < k < 10$  for clear and convincing evidence, and something like  $k \geq 10$  for the reasonable doubt standard. The preponderance threshold is clear, but the two heightened-burden thresholds are theoretically ambiguous, requiring future empirical research.

1. The Preponderance of the Evidence Threshold  
( $k = 1$ )

Deriving the threshold value of the preponderance standard—and of every other standard, for that matter—requires a consideration of the underlying social and legal objectives behind the burden. Most discussion of the preponderance standard is an unhelpful mess of verbal gymnastics<sup>91</sup> and imprecise notions of evidentiary weight.<sup>92</sup> Yet from this general confusion, two clear principles do emerge.

First, the preponderance standard places no special weight on the direction of any mistakes that the fact-finder may make. There seems to be near-universal agreement that, at least in the usual civil suit, the risk of an erroneous factual finding falls no heavier on the plaintiff than it does on the defendant. Justice Harlan provides a typical statement of this normative view in a well-known concurrence: “In a civil suit between two private parties . . . we view it as no more serious in general for there to be an erroneous verdict in the defendant’s favor than for there to be an erroneous verdict in the plaintiff’s favor.”<sup>93</sup> Similar assertions are often made by scholars commenting on the basis for the preponderance standard.<sup>94</sup>

Second, no particular strength of evidence is needed to meet this burden. Sometimes described as the “greater weight of the evidence,”<sup>95</sup> sometimes as facts just “more likely than

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91. See *supra* note 59.

92. See, e.g., MCCORMICK ON EVIDENCE, *supra* note 57, § 339 (noting common judicial instruction against simply adding up the number of witnesses).

93. *In re Winship*, 397 U.S. 358, 371–72 (1970) (Harlan, J., concurring).

94. E.g., Mike Redmayne, *Standards of Proof in Civil Litigation*, 62 MODERN L. REV. 167, 171 (1999) (“[In a typical civil case there] will usually be no reason for valuing the defendant’s rights more than the plaintiff’s rights; consequently, there is no reason for preferring an error in one direction to one in the other.”); MCCORMICK ON EVIDENCE, *supra* note 57, § 341 (making a similar assertion).

95. E.g., *United States v. Matlock*, 415 U.S. 164, 177 n.14 (1974).

not” in the plaintiff’s favor,<sup>96</sup> the inquiry is consistently into the direction that the evidence points, and not into how strongly it points that direction.<sup>97</sup>

These two principles translate into the likelihood theory of fact-finding as a threshold value of  $k = 1$ . Demanding that the likelihood ratio exceed  $k = 1$  for the plaintiff to prevail is a literal translation of the requirement that the scales of evidence tip, however slightly, in favor of the plaintiff.<sup>98</sup> The  $k = 1$  threshold also respects the premise of Justice Harlan, and others, that the preponderance standard should treat the plaintiff and defendant symmetrically. Any other threshold would necessarily give extra weight to the candidate factual theory of one party or the other; only  $k = 1$  affords both sides equal weight in the final inquiry.<sup>99</sup>

An interesting corollary of this threshold value is that it reduces the fact-finding inquiry to a simple search for the most likely factual story on the evidence. Since the most likely factual story is necessarily the candidate theory favoring one of the two parties,<sup>100</sup> the winning story under the preponderance standard is, by definition, the most likely factual story on the evidence.<sup>101</sup> Put another way, this likelihood theory reduces the preponderance of the evidence standard to this question: “What is the most plausible combination of facts on the observed evidence?” The party favored by the answer wins the case.

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96. *E.g.*, *St. Mary’s Honor Ctr. v. Hicks*, 509 U.S. 502, 527–28 (1993).

97. *But cf.* *McBaine*, *supra* note 10, at 249–50 (noting that some courts have tried to distinguish relative weight of evidence from absolute persuasion in this context).

98. *See supra* note 52 and accompanying text. Any value of the likelihood ratio greater than one means that the weight of the evidence falls in the plaintiff’s favor.

99. Some might wonder how this differs from a 51% probability requirement in typical Bayesian analysis. *See supra* note 61 and accompanying text (discussing how level probability statements, like 51%, translate to probability-ratio statements). The difference is that this threshold applies to a ratio of *likelihoods*, not *probabilities*. The considerable implications of that difference are explored in detail in Part III.

100. Since  $C$  and  $C^c$  exhaust all possible combinations of facts in the universe, the most likely combination of facts on the evidence must be either the most likely combination of facts in  $C$  or the most likely combination of facts in  $C^c$ .

101. This assumes unique suprema. Otherwise multiple factual stories might be equally most likely, which complicates discussion of the likelihood theory, but little else.

## 2. The Clear and Convincing Evidence Threshold ( $1 < k < 10$ )

Intermediate burdens like clear and convincing evidence are better understood as a class of standards than as a single entity. For different reasons including social policy, judicial confidence, and special stakes, certain claims and defenses require a party to meet more than a mere preponderance of the evidence to prevail.<sup>102</sup> While asymmetric error tolerance is implicit in these contexts,<sup>103</sup> it is less emphasized than the heightened evidentiary requirement placed on the party bearing the burden.<sup>104</sup>

Building a strength-of-evidence requirement into the likelihood approach is trivial. Any threshold value  $k > 1$  requires the evidence to be at least  $k$  times more likely under the candidate pro-plaintiff factual theory for the plaintiff to prevail. Intuitively, this splits the strength of evidence into three categories: evidence that does not favor the plaintiff ( $LR \leq 1$ ), evidence that weakly favors the plaintiff ( $1 < LR \leq k$ ), and evidence that strongly favors the plaintiff ( $LR > k$ ). The first two categories require finding for the defendant under these intermediate burdens; only strong evidence suffices to justify relief.<sup>105</sup>

None of this says what the numerical value of  $k$  is—and it may well differ by type of case and jurisdiction.<sup>106</sup> The proper threshold value is ultimately an empirical question that I cannot answer with theory alone. But numerical analogies are

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102. See, e.g., MCCORMICK ON EVIDENCE, *supra* note 57, § 340 (noting intermediate standards in a “variety of cases involving deprivations of individual rights not rising to the level of criminal prosecution”); *id.* at 665–66 (noting intermediate standards that reflect inherited rules of fact-finding from courts of equity); *id.* at 668 (noting intermediate standards for claims “disfavored on policy grounds”).

103. See, e.g., *Colorado v. New Mexico*, 467 U.S. 310, 316 (1984) (adopting a clear and convincing evidence requirement to dictate who “should bear most, though not all, of the risks of erroneous decision” but also to “accommodate[] society’s competing interests in increasing the stability of property rights and in putting resources to their most efficient uses”).

104. E.g., MCCORMICK ON EVIDENCE, *supra* note 57, § 339 (commenting that unlike reasonable doubt, the preponderance and clear and convincing formulations direct attention to the evidence); McBaine, *supra* note 10, at 253 (commenting similarly).

105. Obviously, if the defendant bears this burden of persuasion on a particular issue, then the logic is reversed.

106. See MCCORMICK ON EVIDENCE, *supra* note 57, § 340 (discussing variation in the justification for, and articulation of, intermediate standards).

available to assist in finding some empirical calibration for this value. In the cup-choice puzzle, for example, the evidence of a white marble drawn at random from the unidentified cup would be strong enough to satisfy any threshold value,  $k < 5$ . If drawing a white marble in this context feels like clear and convincing evidence that I chose the white cup, then this is one data point in favor of a threshold value of at most 5. If this evidence does not feel clear and convincing, then the exercise may be modified to increase the strength of evidence until it does. There are many exercises to try in this type of calibration process,<sup>107</sup> and in future research rough consensus might be reached on an approximate threshold value. For now, it can only be given as a range.

### 3. The Reasonable Doubt Threshold ( $k \geq 10$ )

It is often claimed that the reasonable doubt standard differs in kind from the preponderance and clear and convincing evidence standards, with the latter two focusing on the weight of the evidence, while the former focuses on the mental state or belief of the fact-finder.<sup>108</sup> This seems more a point of form than substance. The only clear social and legal reason for the distinction between reasonable doubt and these other standards is the special concern about false convictions in the criminal context. Justice Harlan makes this point explicit in the same concurrence as mentioned before: "I view the requirement of proof beyond a reasonable doubt in a criminal case as bottomed on a fundamental value determination of our society that it is far worse to convict an innocent man than to let a guilty man go free."<sup>109</sup> Similar statements have motivated the Court's decisions in other cases involving this standard,<sup>110</sup> and the overarching concern with false convictions is widely

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107. See, e.g., ROYALL, *supra* note 35, at ch. 1 (providing several examples).

108. See, e.g., McBaine, *supra* note 10, at 255 ("In criminal cases the extent or degree of belief of the triers of the fact is stressed, not the amount or quality of evidence."); MCCORMICK ON EVIDENCE, *supra* note 57, § 339 (making a similar observation).

109. *In re Winship*, 397 U.S. 358, 372 (1970) (Harlan, J., concurring).

110. E.g., *Addington v. Texas*, 441 U.S. 418, 423-24 (1979) ("In a criminal case . . . the interests of the defendant are of such magnitude that . . . they have been protected by standards of proof designed to exclude as nearly as possible the likelihood of an erroneous [conviction]. In the administration of criminal justice, our society imposes almost the entire risk of error upon itself").

endorsed in the broader legal community.<sup>111</sup>

At first blush, a focus on false convictions would seem problematic for a likelihood theory of fact-finding. After all, likelihood measures strength-of-evidence, not probability-of-mistake. But there is an intuitive connection between these concepts. We have a general sense that we are more apt to err when acting on weak evidence than when acting on strong evidence—that as evidence becomes overwhelmingly one-sided, the possibility that we are seeing such strong evidence by chance alone becomes less and less plausible.<sup>112</sup> This intuition is borne out in likelihood reasoning, particularly in a general bound that can be placed on the probability of observing evidence that is both strong *and* misleading in a given comparison. When comparing any two hypotheses,  $f'$  and  $f''$ ,<sup>113</sup> where  $f''$  represents the true historic facts, the probability of spuriously seeing strong evidence that misleadingly favors  $f'$  over  $f''$  is bounded not to exceed the inverse of the strength-of-evidence requirement:<sup>114</sup>

$$P\left(\frac{L(f'; e)}{L(f''; e)} \geq k \mid f'' \text{ is true}\right) \leq \frac{1}{k}$$

Put another way, the long-run probability that the evidence would happen to spuriously prove the plaintiff's story declines as the burden of persuasion rises.

This is not exactly a statement about controlling the global probability of false conviction because it cannot always be assumed that either  $f'$  or  $f''$  is true in a given context.<sup>115</sup> But it

111. *E.g.* MCCORMICK ON EVIDENCE, *supra* note 57, § 341 (“Society has judged that it is significantly worse for an innocent person to be found guilty of a crime than for a guilty person to go free . . . [for the] worthy goal of decreasing the number of one kind of mistake—conviction of the innocent.”).

112. One might worry that extremely strong evidence could represent other concerns, such as the destruction of bad evidence or appropriation of false testimony. This is a fair point, but in principle would be built into the weighing of evidence by the fact-finder, and so does not require any special treatment as a theoretical matter.

113. The notation  $f'$  and  $f''$  is read “f prime” and “f double-prime.” This is simply a shorthand way of denoting two different factual hypotheses.

114. ROYALL, *supra* note 35, at 7 (proving and illustrating this universal bound).

115. See Mark L. Taper & Subhash R. Lele, *Evidence, Evidence Functions, and Error Probabilities*, in 7 HANDBOOK OF THE PHILOSOPHY OF SCIENCE 514, 522 (Prasanta S. Bandyopadhyay & Malcolm R. Forster eds., 2011) (discussing the reliability of likelihood-ratio inferences when the true parameter may lie outside

does formalize the intuition that the chances of evidence spuriously favoring conviction tend to fall away as increasingly rigorous evidence is needed to convict.

Defining *error* in the sense of spuriously observing strong evidence for the plaintiff's candidate factual theory when the defendant's theory is really true, the above bound provides a recipe for translating error tolerance into a threshold in the likelihood-ratio test. If as a society we want this long-run rate of error to be no greater than 10%, a threshold value of  $k = 10$  is sufficient. If we want the probability of error to be lower yet, perhaps 5%, then a value of  $k = 20$  is adequate. The long-run rate of error can be made arbitrarily small by demanding an increasingly strong evidentiary showing to convict.

Three caveats are in order. First, these probability bounds are not necessarily tight. While a threshold value of  $k = 10$  guarantees no more than a 10% error rate, the actual frequency of strong and misleading evidence could be substantially lower, perhaps 3%, for example.<sup>116</sup> If so, this bound might accord the criminal defendant more advantage than intended. Second, this is a very narrow definition of wrongful conviction, in which errors are based on the assumption that either the plaintiff's or the defendant's candidate theory is necessarily true. That may well be a reasonable approach, given that we cannot know the actual facts by definition, but it makes the interpretation a little different than a true global bound on the rate of false conviction, and the two concepts should not be confused. Third, care is needed in interpreting the error rate. Evidence of strength  $k = 10$  does *not* mean that there is at most a 10% probability that the evidence is misleading in a given comparison, nor that there is at least a 90% probability of guilt. As already noted, the likelihood approach does not treat guilt or innocence as random in a given case. Instead, the process of using this likelihood-ratio test with this threshold value would—on average, and over the long run—yield no more than 10% strong and misleading comparisons.

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the contrast).

116. Tighter bounds (for which lower threshold values could guarantee the same rate of error) may be derived in specific circumstances where more is known about the relevant probability distributions. See, e.g., ROYALL, *supra* note 35, at 90–94. In a different but related context, Cheng notes that computing the exact probability of false conviction requires very specific information about the probability distributions in question. Cheng, *supra* note 16, at 1277–78.

### C. *Intuition for the Likelihood Approach*

This likelihood theory of fact-finding reduces every burden of persuasion to the same rule of likelihood reasoning. That rule can be further reduced to a four-step algorithm for deciding any case:<sup>117</sup>

- 1) Identify the most likely set of facts in which the plaintiff makes out a cause of action on the evidence.
- 2) Identify the most likely set of facts in which the plaintiff fails to make out a cause of action on the evidence.
- 3) Compare the likelihood of these two factual hypotheses on the evidence.
- 4) Decide for the plaintiff *if and only if* the relative likelihood of the set of facts favoring the plaintiff—the weight of evidence—falls strongly enough in the plaintiff’s favor, meaning:
  - a) Evidence weighs in favor of the plaintiff (in theory, a likelihood ratio greater than 1) for preponderance of the evidence;
  - b) Evidence weighs strongly in favor of the plaintiff (in theory, a likelihood-ratio threshold somewhere between 1 and 10) for clear and convincing evidence;
  - c) Evidence weighs very strongly in favor of the plaintiff (in theory, a likelihood-ratio threshold greater than 10) for the beyond a reasonable doubt standard.

To summarize, the party with the burden of persuasion prevails when the weight of the evidence falls strongly enough in that party’s favor. This is an objective inquiry about the relative consistency of the two competing factual theories with the observed evidence. In no event is the fact-finder required—or even permitted—to base the verdict on what the fact-finder personally believes the facts to be. The question is relative consistency with the evidence, not belief.

This theory of fact-finding differs fundamentally from the conventional Bayesian account. Unlike probability, likelihood is

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117. For simplicity of presentation, this algorithm abstracts from things, like affirmative defenses, for which the plaintiff might lose despite making out a cause of action. The approach can be extended to encompass these nuanced applications.

a purely relative concept of evidential support.<sup>118</sup> In likelihood reasoning, the fact-finder is not forming propositional beliefs about the truth or accuracy of any given combination of facts in isolation. Rather, every step in the process involves the simple comparison of two alternatives in terms of relative consistency with the evidence. This mirrors arguments in the broader evidence literature that juridical fact-finding is a process of exclusively comparative reasoning—not reasoning from the absolute truth of any given factual hypothesis in isolation.<sup>119</sup>

In fact, the proposed likelihood theory can be seen as formalizing many aspects of relative plausibility, narrative coherency, and story-based models of fact-finding, each of which describes some form of iterative comparison of alternatives in an effort to find the *relatively* most plausible story on the observed evidence.<sup>120</sup> But the likelihood theory also adds to these cognitive models of fact-finding. First, it explains how these models scale beyond the preponderance of the evidence setting by providing a strength-of-evidence concept applicable to heightened burdens of persuasion. Second, it justifies what has been a supposed limitation of these models—their focus on the comparison of holistic factual stories rather than element-by-element evaluation against the burden of persuasion. Like

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118. See, e.g., EDWARDS, *supra* note 33, at 28 (noting that probability must be used where an absolute degree of belief in a proposition is needed, but that where relative degree of belief is enough, likelihoods are a sufficient measurement); ROYALL, *supra* note 35, at 8 (“This law [of likelihood] represents a concept of evidence that is essentially relative, one that does not apply to a single hypothesis, taken alone.”).

119. E.g. Ronald J. Allen, *A Reconceptualization of Civil Trials*, 66 B.U. L. REV. 401, 425–28 (1986) (proposing to approach civil trials as a comparative analysis of two competing accounts); Ronald J. Allen, *The Nature of Juridical Proof*, 13 CARDOZO L. REV. 373, 422 (1991) (“There may be cases where cardinal reasoning works, but the typical case calls for ordinal reasoning.”); Cheng, *supra* note 16, at 1259 (“Because the adversarial structure of legal trials promotes jury comparisons of the parties’ claims, preponderance is not an absolute probability . . . [it] is better characterized as a probability ratio, in which the probability of the plaintiff’s story of the case is compared with the defendant’s story of the case.”).

120. See generally sources cited *supra* notes 18–20; see also Daniel Shaviro, *Statistical-Probability Evidence and the Appearance of Justice*, 103 HARV. L. REV. 530, 532 (1989) (“[C]ourts should hold in favor of whichever party appears more likely to be correct.”); Allen, *supra* note 18, at 609 (“In civil cases, fact finders are to accept the more plausible of the stories advanced by the parties, and in criminal cases they are to accept the state’s case only if no plausible story consistent with innocence has been advanced.”); Posner, *supra* note 31, at 1513 (“[The] benchmark for evaluating the plaintiff’s case is not the null hypothesis but the defendant’s case.”).

these models, the proposed likelihood theory of fact-finding involves the comparison of holistic factual stories. The reason it does so is instructive: since the probability of observing any single item of evidence can depend on anything in the entire set of hypothesized facts, there is no general way to separately evaluate isolated facts or elements in a likelihood approach to fact-finding.

A final corollary of this reliance on purely comparative reasoning is that the fact-finder must always consider a no-remedy factual theory in order to decide a case. This does not limit the defendant's right to hold the plaintiff to her proof; nor does it mean that the defendant must put forth a specific theory to prevail.<sup>121</sup> But even if the defendant does not put forth a specific no-remedy theory, the fact-finder must consider one, by independent inference if nothing else. This seems surprising at first—and inconsistent with common notions of how fact-finding works in criminal settings. But it is squarely consistent with the fundamental notion that fact-finding should always result in a *finding of facts*. The final step in this likelihood theory is a choice between two fully specified factual theories; to decide for either party is to find the factual theory favoring that party to be legally true under the applicable burden of persuasion. By definition, this process will result in a single and complete finding of all facts material to a dispute.

### III. COMPARISONS AND IMPLICATIONS

Perhaps the most fundamental thing one might demand of a theory of fact-finding is that it provide sound suggestions for idealized fact-finding—the normative content to say how trial practice and the rules of evidence should handle various situations. Decades of work on probability and belief-based theories have failed to produce a satisfying description of the ideal fact-finding process.<sup>122</sup> But by switching focus away from beliefs

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121. Cf. Cheng, *supra* note 16, at 1262 (“The defendant, particularly in a civil case, may not simply be a contrarian. The jury expects the defendant to present an alternative view of the evidence, and so like the plaintiff, the defendant too must present an explanation of what happened. To the extent that civil trials are about factfinding or truth, it will not do for the defendant’s theory to be ‘plaintiff’s story.’”).

122. Compare Allen, *supra* note 119, at 402–03 (noting problems with probability theories of fact-finding in the 1980s), with Allen & Stein, *supra* note 32, at 560–65 (noting the same problems and paradoxes in modern probability

about the probabilistic truth of facts and toward the relative consistency of factual hypotheses with the evidence, the proposed likelihood theory of fact-finding fares far better. This part highlights some preliminary implications of this change in focus, addressing two questions in particular:

- Which approach—probability or likelihood—is the more plausible description of the fact-finding process?
- What normative implications does this new likelihood theory have for trial practice and the rules of evidence?

The answers to these questions touch everything from the mechanical process of the ideal fact-finder, to the language of jury instructions, to the justification of evidentiary resistance to character and propensity reasoning and naked statistical evidence, to the conceptual framework for explaining how prejudice and personal bias affect fact-finding and why just fact-finding cannot abide their influence.

#### A. *Implications for the Ideal Fact-Finding Process*

To show how the proposed likelihood theory differs from probability and belief-based models of the ideal fact-finding process, it suffices to survey a few of the difficulties inherent in probability and belief-based models. Indeed, even the basic mechanics of a Bayesian theory of fact-finding prove surprisingly hard to pin down. The problem is that all probability statements describe something about the *total* uncertainty in a system; thus, the probabilities of all possible alternatives must sum to one. To see the headaches this creates for a probability or belief-based description of idealized fact-finding, consider even the lowly preponderance of the evidence standard.

The traditional probability-based articulation of the preponderance standard states that the plaintiff must prove “the existence of [any material] fact [to be] more probable than its nonexistence.”<sup>123</sup> Put more formally, the plaintiff is required to prove that the probability of a combination of facts in the cause-of-action set is greater than 50% or—equivalently—that the probability that this combination of facts is true is greater

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theories of fact-finding).

123. Flemming James Jr., *Burdens of Proof*, 47 VA. L. REV. 51, 54 (1961); see also McBaine, *supra* note 10, at 260–61 (suggesting a similar rule).

than the negation of that claim.<sup>124</sup> The suggestion is that the idealized fact-finder would assess the posterior probability of the plaintiff's theory on the evidence, then assess the probability of all other possible combinations of facts on the evidence, and then compare the ratio of these probabilities to one to see if the plaintiff wins.

This leads to shockingly absurd results. Suppose two cars collide on a 25-mph road, and that the plaintiff's theory includes a claim that the defendant was driving 60 mph at the time of the collision. The negation of this claim includes every other speed that the defendant could have been driving—including still-dangerous speeds (e.g., 59.5 mph) as well as even more dangerous speeds (e.g., 70 mph). A strict application of the above test would count the probabilities of these strongly pro-plaintiff alternative facts *against* the plaintiff's right to recovery.

In fairness, this is a bit of a straw man: no advocate of probability reasoning would endorse the literal application of the traditional test, or the standard jury instruction that accompanies it. Instead, it might be argued that the plaintiff is really alleging a composite set of alternative facts to support recovery. Perhaps the composite fact is that the defendant was driving "over the speed limit" in the prior example. But this only shifts the problem to a different place. Now, the possibility that the defendant was not speeding but was instead drunk or distracted with the radio counts against the plaintiff's right of recovery in this idealized model of the fact-finding process.

The only way that the aggregation of alternative factual theories fully escapes these bizarre results is for the idealized fact-finder to consider every possible combination of facts in the entire cause-of-action set. I am unaware of any common jury instructions to this effect, which is gratifying given the magnitude of the task it contemplates. The idealized fact-finder would need to (1) identify every combination of facts in the entire cause-of-action set; (2) assess the absolute probability of

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124. Ronald Allen, *On the Significance of Batting Averages and Strikeout Totals: A Clarification of the "Naked Statistical Evidence" Debate, the Meaning of "Evidence," and the Requirement of Proof Beyond a Reasonable Doubt*, 65 TUL. L. REV. 1093, 1093 (1991) ("The conventional conception of civil trials involves comparing the probability of a plaintiff's case to its negation."); Cheng, *supra* note 16, at 1258 n.10 ("[C]onventional legal treatments focus on the likelihood ratio between the plaintiff's story being true and the plaintiff's story being false (as opposed to the defendant's story being true).").

every combination of facts in this possibly infinite set; and (3) sum up all of these probabilities in arriving at a single, aggregate probability or belief measure to be compared against the burden of persuasion.

As a description of idealized trial fact-finding, this is wanting. First, the scope of the task feels out of place in the typical trial context. When—if ever—would a trial map out the absolute probability of every possible combination of facts in the entire cause-of-action set?<sup>125</sup> Second, the approach leads to a peculiar concept of fact-finding. Suppose that each of six alternative factual theories is found to have only a 10% probability of being true. Together, the aggregate probability of these six alternative theories satisfies the preponderance standard and entitles the plaintiff to relief. But why? What happened to justify relief? That is, what *facts* has the fact-finder *found* in this situation?

Seeking to avoid these problems, some scholars have abandoned aggregation strategies in favor of descriptions of idealized fact-finding based on the probability ratios of individual pairs of factual theories.<sup>126</sup> Cheng provides a recent example. Though continuing to interpret fact-finding in terms of Bayesian probabilities, Cheng imposes the ad hoc assumption that fact-finders cannot aggregate the probabilities of alternative factual theories together, leaving only the comparison of individual factual theories as a basis for inference.<sup>127</sup> If the plaintiff and defendant each advance one factual theory, Cheng's proposed model of idealized fact-finding defines the plaintiff's right to recover by the degree to which the posterior probability of the plaintiff's theory exceeds the posterior probability of the defendant's theory. That is, the idealized fact-finder would compare the posterior probabilities of *only* these two individual factual hypotheses in determining the winner.

But ad hoc modification of the axiomatic properties of probabilities is an odd way to proceed in a theory based on probabil-

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125. Cf. James, *supra* note 123, at 52 (“[Our adversarial system] frees the judge and jury of responsibility for investigating and presenting facts and arguments, placing that responsibility entirely upon the respective parties . . .”).

126. Allen, *supra* note 119, at 425–28 (providing what appears to be the first formal suggestion of this approach).

127. Cheng, *supra* note 16, at 1262 (“The defendant, particularly in a civil case, may not simply be a contrarian . . . . The defendant may offer multiple possible alternatives, but each of these alternatives will be judged separately, not simultaneously.”).

ities.<sup>128</sup> And, here, it creates nearly the opposite paradox of the previous models. Suppose the critical fact in a negligence action is whether the defendant ran a red light. The plaintiff claims the light was red; the defendant claims it was green. If the fact-finder concludes that the posterior probability of a red light on the evidence is  $P(R | e) = 0.4$  and that the posterior probability of a green light is  $P(G | e) = 0.3$ , then on Cheng's theory the plaintiff wins and the defendant is liable. But since probabilities must always sum to one, the above conclusions necessarily imply the probability of a yellow light is  $P(Y | e) = 0.3$ . Cheng's theory would assign liability under circumstances that actually prove the light was probably *not* red at all:  $P(G \text{ or } Y | e) = 0.6$ .

All of these problems arise from reliance on absolute probability or belief as the measure of legal fact-finding. In changing the concept of uncertainty from probability to likelihood—the theoretical analog of relative plausibility or relative consistency of different factual hypotheses with the evidence—all of these problems disappear:

- Compared to the traditional probability model, close alternatives to the candidate hypotheses do not enter any part of the likelihood ratio. If the most likely pro-plaintiff theory is that the defendant was going 60 mph, then the existence of less likely theories (like 59 mph) has no bearing on the fact-finder's ultimate decision.
- Compared to aggregate probability models, the likelihood theory involves a series of purely relative comparisons. And in theory, it will always conclude in a single, fully specified finding of facts.
- Compared to probability ratio models, likelihood analysis requires no modification of the intrinsic properties of likelihoods. The yellow-light paradox does not apply, as likelihoods do not sum to one (meaning that finding the evidence more consistent with a red light than a green light needn't convey any information at all about the relative consistency of the evidence with a yellow light).<sup>129</sup>

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128. Cf. Allen & Stein, *supra* note 32, at 596 (“Mathematical probability is a system of reasoning that one must either use in its entirety or not use at all. There is no room for picking and choosing.”).

129. For a discussion of the alternative possibility that the fact-finder views the set of possible facts as binary (was the light *red* or was it *not-red*?), see *infra* Part IV.

Instead, likelihood analysis supports the simple model of idealized fact-finding summarized in Section II.C, and described in greater detail in Section II.B. This process—which involves only the iterative comparison of alternative factual stories on the available evidence—closely mirrors the typical presentation of evidence in adversarial litigation, as well as empirical accounts of the actual deliberative process of fact-finders.<sup>130</sup>

### B. *Implications for the Conjunction Paradox*

Another view of how a likelihood theory of fact-finding differs from a probability or belief-based theory is afforded by comparing how each approach conceptualizes facts in the fact-finding process. As already noted, the likelihood model typically assumes that facts are immutable constants. Examples include the historic actions of the parties, or a person's true mental state at the time of a transaction. By contrast, probability models of fact-finding treat the facts of a case as random variables, implying that the facts must obey the laws of probability. This, in turn, leads to predictably strange results.

A modest example is a complication arising from the property that continuous random variables have infinitesimal probability of taking on any given value. To illustrate, suppose a factual theory includes the claim that the defendant was driving 60 mph on a particular highway. The probability that the defendant was really going exactly 60 mph is technically zero. Even if this is a close estimate, the truth might be 61 mph, 59.5 mph, or 60.001 mph. With enough evidence, any single guess can almost surely be disproved. This technical oddity muddies the model. It means that the probability of any combination of facts that includes a continuous variable (like driving speed) can never exceed 50%, or even 0% for that matter. The problem is mitigated by aggregating probabilities of alternative factual theories, but that has its own problems, as discussed in the previous subsection.

Another—and far more troubling—complication arising from the treatment of facts as random variables is what the literature has come to call the *conjunction paradox*.<sup>131</sup> In brief,

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130. See *supra* notes 18–20, 119–120 and accompanying text.

131. See Mark Spottswood, *Unraveling the Conjunction Paradox*, 15 LAW,

the conjunction paradox is the unsettling discovery that when material facts are treated as random variables, the threshold probability needed to prove any given fact (or element of a cause of action) may diverge from the threshold probability needed to meet the burden of persuasion on the overall claim for relief.<sup>132</sup> A thought experiment illustrates the problem.

Suppose a case with two disputed facts is being tried under the preponderance standard. To keep things simple, each fact is either true or false, and the plaintiff wins only if both facts are true. At the close of evidence, the fact-finder concludes that the posterior probability of each fact being true is  $P(f_1 | e) = 0.7$  and  $P(f_2 | e) = 0.7$ . Each fact is thus more probably true than false, which seems like it would satisfy the usual jury instruction on the preponderance standard.<sup>133</sup> But the probability of *both* facts being true may actually be much smaller than either individual probability. For example, if  $f_1$  and  $f_2$  are independent variables (such that the truth or falsity of one fact suggests nothing about the truth or falsity of the other), then the probability of both facts being true is  $P(f_1 \text{ and } f_2 | e) = 0.7 \times 0.7 = 0.49$ , which means that the plaintiff has actually failed to carry the burden of persuasion.

The conjunction paradox is a serious problem for any probability theory of fact-finding. First, it complicates the description of the burden of persuasion, since the threshold probability for finding individual facts differs from that needed to decide the overall case, and actually depends on things—like the number of disputed issues—that vary from one case to the next.<sup>134</sup> Second, it implies that the actual difficulty of satisfying a burden of persuasion depends on the number of elements or material facts in a claim.<sup>135</sup> To illustrate this issue,

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PROBABILITY & RISK 259, 259 n.2 (2016) (collecting a large number of important works on this topic).

132. *Id.* at 259–60.

133. *See supra* note 50 and accompanying text.

134. *Cf.* Posner, *supra* note 31, at 1513–14 (oscillating on how a fact-finder would decide the case when individual and joint probabilities lead to different outcomes).

135. *E.g.*, Sanchirico, *supra* note 11, at 222 (“Particularly troubling is the fact that the implied threshold probability for a charge, claim, or defense decreases (quite rapidly) in the number of elements it contains, a factor with uncertain relevance.”); Allen, *supra* note 119, at 406–07 (“One implication of the conjunction principle is that it injects a certain inequality of treatment into the trial of disputes that is a function of the number of elements of a cause of action . . . plaintiffs’ tasks will become more difficult as each new independent element is

note that theft has more elements than murder in most jurisdictions, yet few see a viable argument for subjecting the elements of theft to more a searching evidentiary standard.<sup>136</sup> Third, it suggests that the defendant's mere act of disputing an additional issue should have the surprising effect of increasing the plaintiff's effective burden of persuasion not just on that issue, but on all other issues as well.<sup>137</sup>

Few real attempts have been made to save the probability approach from the conjunction paradox. For example, it might be argued that the burden of persuasion should be applied to individual issues without regard to the probability of their joint truth.<sup>138</sup> Or that alternative factual theories should not be aggregated, so that the 0.49 probability of both facts being true can only be compared to the individual probabilities of just  $f_1$ , just  $f_2$ , or neither being true (0.21, 0.21, and 0.09, respectively).<sup>139</sup> But both of these arguments amount to ad hoc modification of the rules of probability in a theory meant to derive from the rules of probability—a strong hint that probability concepts are an awkward way to understand uncertainty in legal fact-finding.

Likelihood reasoning avoids all random-facts problems, including the conjunction paradox. It does so by not treating the facts as random in the first place. Instead, likelihood analysis generally treats the facts of a case as fixed, and the *evidence* as random. It then compares the probability of seeing the evidence under different factual hypotheses. The source of the conjunction paradox—the product rule of probabilities—still applies in likelihood analysis, but it applies to the evidence, not the facts.

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added.”).

136. Leiter & Allen, *supra* note 15, at 1504–05.

137. See Cheng, *supra* note 16, at 1263 (“It seems odd, however, that merely disputing another element of the tort not only creates a burden on the plaintiff regarding that element, but also raises the standard by which the plaintiff must prove [all other] elements at issue.”).

138. E.g., Alex Stein, *An Essay on Uncertainty and Fact-Finding in Civil Litigation, with Special Reference to Contract Cases*, 48 U. TORONTO L.J. 299, 311–12 n.27 (1998) (arguing that the conjunction paradox could be avoided by basing outcomes on the probabilities of elemental issues and ignoring their joint probability). Stein has since denounced this argument. See Allen & Stein, *supra* note 32, at 595–96.

139. Cheng, *supra* note 16, at 1263–65. Cheng characterizes this solution as a result of comparing probability ratios, *id.* at 1263, but the actual argument around the conjunction paradox rests on an assumption that the defendant cannot aggregate probabilities of alternative facts to rebut the plaintiff's case, *id.* at 1264.

And since the same evidence appears on the top and the bottom of the likelihood ratio, the product rule does not present the same asymmetry in likelihood reasoning that it does in probability reasoning.

Intuitively, likelihood reasoning treats the facts of the case as the conditioning parameters in probability statements. Adding disputed facts changes the way the fact-finder thinks about the various factual hypotheses and their consistency with the evidence, but does not have any general effect on the ease or difficulty of meeting the burden of persuasion. Two implications are apparent from this result: (1) it is generally inappropriate to describe burdens of persuasion as applying to individual elements of a cause of action, and (2) when approached in likelihood terms, fact-finding exhibits no conjunction paradox.

### C. *Prior Probability, Character Reasoning, and Naked Statistics*

A third useful comparison between probability and likelihood theories of fact-finding is afforded by the different ways these theories suggest that fact-finders should use prior probabilities in their deliberations. To keep things concrete, this discussion focuses on two infamous puzzles in the evidence literature: the *Gatecrasher* and the *Blue Bus* paradoxes.

The Gatecrasher paradox is a toy fact-pattern that leads to a strange result under traditional probability reasoning.<sup>140</sup> Suppose one thousand people are in the stands of a rodeo, but a look in the cash register shows that only 499 of them have paid the price of admission. Nothing indicates who paid and who didn't, but it is clear that 501 people have jumped the gate. It has been noted—many times over—that the probability that a randomly chosen attendee would be one of the gate jumpers is 50.1%, making any randomly chosen attendee liable for the price of admission under the preponderance standard of a Bayesian probability model of fact-finding. Everyone agrees that Bayesian logic compels this result, but no one seriously thinks it is the right outcome.<sup>141</sup>

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140. See L. JONATHAN COHEN, *THE PROBABLE AND THE PROVABLE* 75 (1977) (giving the first description and illustration of the Gatecrasher paradox).

141. *E.g.*, Allen & Stein, *supra* note 32, at 573–74 (“[The attendee’s] claim that he actually paid for his admission to the rodeo only has a 0.499 probability.”)

The Blue Bus paradox is a slightly different situation, loosely based on the facts of an actual case.<sup>142</sup> The plaintiff is driving home at night when a reckless bus driver forces her off the road and into a ditch. The plaintiff has time to note only the color of the bus—blue. She sues the local Blue Bus Company, alleging that it owns 80% of all blue buses in the area. None of this is contested, and no other evidence is put up by either side. Many commentators conclude that this means the Blue Bus Company has an 80% probability of responsibility.<sup>143</sup> But, again, few are comfortable assigning liability on this record alone.<sup>144</sup>

Efforts to defend the Bayesian probability approach against these paradoxes have been strained and unpersuasive. A common argument is that the fact-finder can escape the requisite liability conclusion by drawing a negative inference from the absence of better evidence in both puzzles.<sup>145</sup> But this is tantamount to assuming away the problem. Other arguments suggest that auxiliary policy objectives—like judicial economy or process validity—may require the plaintiff to show more than technically needed to win in these scenarios.<sup>146</sup> Perhaps, but this is again dodging the core difficulty of each puzzle. Instead, the clearest way to reconcile the result in each of these puzzles with our intuition is to say that the *paradox* in each puzzle comes from the effort to base fact-finding conclusions on

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Hence, under the preponderance standard . . . the organizers appear to be entitled to recover [from the randomly chosen attendant], which is patently absurd.”)

142. *Smith v. Rapid Transit, Inc.*, 58 N.E.2d 754 (Mass. 1945). The hypothetical given in the text reflects Laurence Tribe’s stylized version of this case. Tribe, *supra* note 22, at 1341–42 n.7.

143. The standard Blue Bus hypothetical actually provides insufficient information to compute an unambiguous probability of responsibility. *Cf. infra* note 150.

144. *E.g.*, Posner, *supra* note 31, at 1508–09 (interpreting a similar hypothetical in a way that implies the Blue Bus Company would have a posterior probability of liability of 80% and noting the intuitive absurdity of this result, at least in cases where the posterior probability of liability is not too great).

145. *E.g.*, David Kaye, *The Paradox of the Gatecrasher and Other Stories*, 1979 ARIZ. ST. L.J. 101, 107–08 (1979) (arguing that the fact-finder may draw an inference from the lack of other information in the hypothetical); Posner, *supra* note 31, at 1509 (“The problem that causes this disbelief, however, is not with mathematical probability but with the tacit assumption that the statistic concerning the ownership of the buses is the only evidence that the plaintiff can obtain.”).

146. *E.g.*, Posner, *supra* note 31, at 1509 (arguing that judicial economy may require the plaintiff to expend more effort in these cases); *see also* Wells, *supra* note 63, at 740 (noting and citing various similar policy arguments).

prior probabilities.

This is easiest to see in the Gatecrasher paradox. Consider how the situation would be formalized under Bayes' Theorem for a randomly chosen rodeo attendee:

$$\frac{P(\text{jumped} | e)}{P(\text{paid} | e)} = \frac{P(e | \text{jumped})}{P(e | \text{paid})} \times \frac{P(\text{jumped})}{P(\text{paid})}$$

The posterior probability ratio of jumping to paying (left term) equals the likelihood ratio for the evidence (middle term), multiplied by the prior probability ratio of jumping to paying in the overall group of attendees (right term). The likelihood-ratio equals precisely one in this puzzle: the only "evidence" is the contents of the ticket box, and the contents of this box would be identical whether the defendant was one of the paying attendees or one of the gate jumpers. Rather, this is the rare case of a clear prior probability: there is a 50.1% chance that a randomly chosen attendee would be one of the gate jumpers. Thus, the posterior probability ratio equals a likelihood ratio of one times a prior probability ratio of  $50.1/49.9 = 1.004$ , proving any randomly chosen attendee liable under the preponderance standard of a Bayesian probability theory of fact-finding.

This liability determination is entirely driven by prior probabilities, and discomfort with the outcome belies an intuitive resistance to assigning guilt or liability on a prior-probability basis. The principles of likelihood reasoning comport with that hesitancy. In fact, likelihood analysis of this puzzle does not consider prior probabilities at all. With a likelihood ratio of exactly one, there is not enough evidence (literally "no evidence") to meet the preponderance standard of the likelihood theory of fact-finding:

$$\frac{L(\text{jumped}; e)}{L(\text{paid}; e)} = \frac{P(e | \text{jumped})}{P(e | \text{paid})} = 1$$

In contrast to the paradoxical result reached by Bayesian probability reasoning, a randomly chosen attendee would not be held liable under a likelihood approach to this puzzle.

This difference in outcomes reflects a fundamental difference in the way each theory treats information about average behavior and background frequencies. As explained above, Bayesian probability analysis expects the fact-finder to base

decisions—at least in part—on the prior probabilities of material facts. Here, the ticket box that describes the average behavior of all the rodeo attendees is used to reason about the culpability of a single, randomly chosen attendee. Likelihood analysis does not involve prior probabilities in any stage of the fact-finding process. The average number of paying customers in the stands is irrelevant to a likelihood analysis of the puzzle. To see just how far likelihood reasoning goes in ignoring prior probabilities, consider the following two variations on the usual Gatecrasher puzzle.

First, suppose that only one ticket is missing from the ticket box, but a ticket agent takes the stand to testify that she thinks she saw the defendant jump the gate. This testimony is hardly strong evidence of the defendant's liability, but if that's all there is—and if the testimony is given full credit by the factfinder and not, for example, assumed to be a self-serving lie—then it alone suffices to carry the preponderance standard under likelihood reasoning. It does not matter that only one attendee in the crowd jumped the gate. The legal inquiry is about *this* attendant, and the only available evidence points toward liability.<sup>147</sup>

Second, suppose there is no testimony to be had, but a look in the ticket box reveals one lonely ticket—the implication being that fully 999 out of the one thousand attendees have jumped the gate, making the prior probability 99.9% that a randomly selected attendee would be a gate-jumper. Just like the standard Gatecrasher puzzle, this version of the puzzle presents no evidence in the likelihood sense of the term. One attendee has paid the price of admission, and nothing has been shown to indicate that this particular attendee is more likely a jumper than the (one) payer. Thus, even in this extreme setting, likelihood reasoning would not find a randomly chosen at-

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147. Some might puzzle over why the inquiry is between crowd members, and not between a crowd member and someone in the larger public: with even a single ticket missing, the likelihood that a randomly chosen attendee is a gate jumper will always exceed the likelihood that someone outside the stadium is a gate jumper. This simply reflects the fundamental problem that the framing of fact-finding matters. By emphasizing the importance of framing effects, the likelihood theory may help to clarify what the correct frame of reference actually is in the fact-finding process. Here, for example, the relevant legal question seems to be *which* of the attendees jumped the gate (the between-attendee comparison), not *whether* any of them did (the between-attendee-and-non-attendee comparison).

tendee liable for the price of admission.<sup>148</sup> Of course, social policy might suggest holding all members of the group jointly liable in such an extreme situation.<sup>149</sup> But that only clarifies the critical point: available information proves the group liable, not any particular individual therein.

Similar logic applies in the Blue Bus case. The clearest way to frame this puzzle is as a prior probability ratio of 80%/20% in favor of the Blue Bus Company's responsibility, again with a likelihood ratio of exactly one.<sup>150</sup> As in the Gatecrasher puzzle, lack of likelihood-relevant evidence prevents a finding of liability under likelihood analysis.

But consider a related puzzle proposed by Gary Wells: the same circumstances as before, but instead of the Company owning 80% of the blue buses, a weigh-station logbook records a Blue Bus Company bus passing down the road just before the collision—although, on cross-examination, the logbook is also shown to be accurate only 80% of the time.<sup>151</sup> This revised puzzle contains likelihood-relevant evidence. The logbook has an 80% probability of correctly identifying the Blue Bus Company if a Company bus was responsible for the incident, and only a 20% probability of falsely identifying the Blue Bus Company if the offending bus did not belong to the Company ( $LR = 0.8/0.2 = 4$ ). A likelihood approach to fact-finding assigns liability in the logbook version of the puzzle ( $LR = 4$ ), but not the canonical

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148. This is a knife-edge result, since a ticket box without a single ticket in it would conclusively prove every attendee liable. This discontinuity may initially seem odd, but as explained in the remainder of the paragraph, it makes sense when approached from the perspective of an inquiry about an individual defendant. Similar discontinuities already exist in the law of evidence. *Compare* FED. R. EVID. 404(b)(1) (stating the usual prohibition on the use of evidence of prior bad acts to infer repetition of the bad act on a particular occasion), *with* 1 MCCORMICK ON EVIDENCE § 190(2) (Kenneth S. Broun ed., 7th ed. 2013) (summarizing the usually permitted use of evidence of highly distinctive prior acts—*modus operandi*—to infer repetition of the act on a particular occasion).

149. *See, e.g.*, *Summers v. Tice*, 199 P.2d 1 (Cal. 1948) (providing the classic example of alternative liability, in which proof that some member of a group caused an injury shifts the burden to the group members to prove their individual innocence).

150. Not enough information is provided to be certain how to frame the Blue Bus case. Depending on assumptions about the prudence of drivers and the frequency of other bus colors, scenarios consistent with the puzzle can range from strong proof of liability, to strong proof against liability. To illustrate, consider the puzzle-consistent situation where the Blue Bus Company has 8 blue buses and 990 red buses, while the only other bus company in the area has 2 blue buses and nothing else.

151. Wells, *supra* note 63, at 741.

version ( $LR = 1$ ).<sup>152</sup> Lack of individualized evidence saves the Blue Bus Company in the canonical version, just as the availability of individualized evidence condemns it in the logbook version of the puzzle.

These examples illustrate likelihood reasoning's sensitivity to the type of information being considered. Likelihood-relevant evidence is used; prior probability is not. Admittedly, the distinction between these concepts is not always great, and whether some item of information characterizes evidence or prior probability can be sensitive to the framing of fact-finding questions.<sup>153</sup> But while resting a sharp distinction on so subtle a difference may seem like a defect of the likelihood theory of fact-finding, it may actually be the theory's greatest strength.

First, whether ideal or not, the sharpness of this distinction reflects empirical realities. The whole reason that the Gatecrasher and Blue Bus puzzles are paradoxes in the first place is that the conclusions of Bayesian probability analysis are at odds with our intuition about how these cases should turn out. Applying likelihood analysis to the puzzles yields conclusions aligned with our intuitive expectations. This suggests that fact-finding may already involve likelihood reasoning—and it is not the only evidence to that effect.

In comparing the canonical- and logbook-versions of the Blue Bus puzzle in a series of psychology experiments, Wells finds that subjects assess the same posterior probabilities in both versions of the puzzle, yet assign liability frequently in the

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152. How is an ownership proportion of 80% different from the testimony of a witness who is correct only 80% of the time? One way to consider the difference is to ask how the available information would differ if the Blue Bus company were *not* responsible. It would obviously own 80% of the blue buses either way, but the eyewitness's testimony has only a 20% chance of falsely identifying the Blue Bus company if that company was innocent. Information that depends on the underlying facts is likelihood-relevant evidence. *See id.* at 746 (offering a similar interpretation under the rubric of "fact-to-evidence reasoning" as distinct from "evidence-to-fact reasoning").

153. Suppose a negligence case arises from a traffic collision. The defendant argues that she was driving carefully when her brakes randomly failed. The plaintiff replies with information on the rarity of random brake failure. One view of the plaintiff's showing is that it describes the prior probability of the defendant's theory. Another view is that this showing helps the fact-finder to contrast the likelihood of seeing the evidence (a traffic collision) under the competing theories that the defendant was driving negligently (for which a crash is relatively probable) as opposed to driving prudently (for which a crash from random brake failure is relatively improbable). *See infra* Part IV for a more complete discussion of this scenario.

logbook version of the puzzle, but rarely in the canonical version.<sup>154</sup> This is inconsistent with fact-finding based on Bayesian probability reasoning: subjects assess the same posterior probabilities in both versions of the puzzle but reach different results at the fact-finding stage. It is entirely consistent with likelihood reasoning—and with subjects demanding individualized, likelihood-relevant evidence to support a finding of liability.<sup>155</sup>

Second, the distinction between individualized, likelihood-relevant evidence and prior probability reflects existing concepts in the law of evidence. One clear example is the usual ban on character reasoning. Evidence of a person's character (or propensity to act some way) is generally inadmissible to prove that the person did act in conformity with that character (or propensity) on a particular occasion.<sup>156</sup> This is not an obscure or technical evidentiary exclusion. It is a fundamental rule that codifies at least one aspect of the prior-probability distinction in likelihood analysis by prohibiting the fact-finder from using character-based prior probabilities in deciding how a person actually acted on a particular occasion.<sup>157</sup>

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154. Wells, *supra* note 63, at 742 fig.1; *see also id.* at 744 fig.3; *cf. id.* at 743–44 (also describing a third treatment in which the provided information could either be framed as likelihood-relevant evidence or prior probability information, but which subjects generally treated the same as the canonical version of the paradox).

155. *Id.* at 746 (“The hypothesis offered here is that in order for evidence to have a significant impact on people’s verdict preferences, one’s hypothetical belief about the ultimate fact must affect one’s belief about the evidence.”); *id.* at 750 (“[I]t could be argued that people will allow their subjective probabilities to drive their verdict decisions only if the evidence on which those subjective probabilities are based is responsive to assumptions about the ultimate fact.”).

156. *E.g.*, FED. R. EVID. 404; *cf.* Paul F. Rothstein, *Intellectual Coherence in an Evidence Code*, 28 LOYOLA L.A. L. REV. 1259 (1995) (discussing and critiquing theories of the justification for this doctrine and its exceptions); Peter Tillers, *The Death of a Youth and of a Drunkard: A Remarkable Story of Habit and Character in New Jersey*, in EVIDENCE STORIES 29, 30 (Richard Lempert ed., 2006) (“[G]uilt or innocence is supposed to depend on what a person does, and not on what a person is.”).

157. One might reason that rules of evidence allowing the use of evidence of habitual acts represent a counter-argument. *See, e.g.*, FED. R. EVID. 406 (“Evidence of a person’s habit . . . may be admitted to prove that on a particular occasion the person . . . acted in accordance with the habit . . .”). But this, too, is consistent with likelihood analysis. The reason is another subtle matter of framing. The prohibited act in character reasoning is to infer from evidence of a person’s propensity to act some way (prior probability) something about the conscious and willful (non-random) acts of that person. By contrast, the type of habitual-act evidence that is most clearly admissible is that describing the nearly

Another example is judicial hostility to naked statistical evidence. When faced with purely statistical evidence, courts and fact-finders often refuse to rely upon statistics that are not in some way individualized to the specific parties or transaction at issue in the case.<sup>158</sup> An old state court opinion colorfully captures the tenor of this thinking:

That in one throw of dice there is a quantitative probability, or greater chance, that a less number of spots than sixes will fall uppermost is no evidence whatever that in a given throw such was the actual result . . . . The slightest real evidence that sixes did in fact fall uppermost would outweigh all the probability otherwise.<sup>159</sup>

This demand for individualized evidence over mere probability is bewildering from a Bayesian perspective because individualized evidence (likelihood-relevant evidence) and statistical averages (prior probabilities) both factor into computing posterior probabilities.<sup>160</sup> But judicial insistence on individualized evidence is easily explained by the likelihood theory of

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automatic act of a person. See, e.g., FED. R. EVID. 406 advisory committee's cote to the 1972 proposed rules ¶ 2 ("The doing of the habitual acts may become semi-automatic . . ."). These semi-automatic acts can be appropriately understood as random processes: facing certain specific conditions, the person exhibits a pattern response with a low probability of deviation. In the specific case where the fact at issue is itself an explicitly random process, likelihood analysis has no objection to prior probabilities. This is not a defect of the likelihood model, but a recognition of the different type of uncertainty inherent in random processes. For further discussion of the topic of random facts, see *infra* Part IV.

158. E.g., *United States v. Shonubi*, 103 F.3d 1085, 1092 (2d Cir. 1997) (distinguishing "specific evidence" of the defendant's conduct from background statistics on what "117 other people had done" under similar circumstances); cf. Allen, *supra* note 124, at 1099 ("If a statistic has no counterfactual implications, if it really is just an accidental property, then it tells us nothing about an event that is not in the particular set that generated the statistic.").

159. *Day v. Boston & M.R.R.*, 52 A. 771, 774 (Me. 1902).

160. See Jonathan J. Koehler, *The Normative Status of Base Rates at Trial*, in *INDIVIDUAL AND GROUP DECISION MAKING: CURRENT ISSUES* 137, 141 (N. J. Castellan, Jr. ed., 1993) (relegating the idea that prior probabilities are irrelevant to fact-finding, because they only describe group or long-run behavior, to the status of an argument "more likely to be advanced by law students and practicing attorneys who have little or no familiarity with statistics or probability theory"); see also Peter Tillers, *If Wishes Were Horses: Discursive Comments on Attempts to Prevent Individuals from Being Unfairly Burdened by Their Reference Classes*, 4 *LAW, PROBABILITY & RISK* 33, 36 (2005) (presenting and discussing an argument "that the . . . distinction between specific and non-specific [statistical] evidence is almost unintelligible").

fact-finding. The distinction between evidence and probability seems to be no more and no less than the distinction between likelihood-relevant evidence and prior probability. In fact, efforts to define naked statistical evidence provide a useful rule of thumb for identifying information that characterizes prior probabilities: irrelevant prior probability information is “not case specific in the sense that the evidence was not created by the event in question but rather existed prior to or independently-of the particular case being tried.”<sup>161</sup>

Third, to the extent that the sharp distinction between likelihood-relevant evidence and prior probability reflects common fact-finding practice, it is important that a theory of fact-finding reflect this distinction. While it may often be clear what information constitutes likelihood-relevant evidence, as opposed to a prior probability, some cases will inevitably fall close to the line. And in those cases, it is better that the legal community recognize, understand, and struggle to enforce the difference between prior probability and likelihood-relevant evidence than it is for the whole distinction to be swept under the rug as it is in a Bayesian understanding of fact-finding.

#### *D. Personal Beliefs, Prejudice, and Biased Deliberations*

A final distinction to highlight between a probability or belief-based approach to fact-finding and a likelihood approach is the different tolerance each theory has for subjective beliefs and fact-finder bias. Because a probability theory of fact-finding relies on the fact-finder to supply his or her own prior probabilities for all material facts, it necessarily and explicitly builds subjective prior beliefs, prejudice, and bias into its account of the idealized fact-finding process. Likelihood reasoning contemplates no such thing. Idealized fact-finding in the proposed likelihood theory is as free of subjective beliefs as it can be. The likelihood theory provides a framework for understanding the harm of using prior beliefs in fact-finding, and also a normative argument—as if one were needed—for further efforts to eliminate subjective prior belief, prejudice, and bias from the fact-finding process.

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161. Wells, *supra* note 63, at 739 (providing this as a typical definition for “[n]aked statistical evidence,” but noting that the term is ill-defined in the legal literature).

To see how deeply and distressingly prior beliefs shape fact-finding in a probability or belief-based approach, consider the translation of a Bayesian model of the fact-finding process into likelihood terms.<sup>162</sup> Suppose that there are only two factual theories at issue. The first,  $f' \in C$ , is the plaintiff's theory of the case; the second,  $f'' \in C^c$ , is the defendant's theory. In a Bayesian approach, an idealized fact-finder would find for the plaintiff only if the posterior probability of the plaintiff's theory sufficiently exceeded that of the defendant's theory:<sup>163</sup>

$$PR = \frac{P(f'|e)}{P(f''|e)} = \frac{L(f';e)}{L(f'';e)} \times \frac{P(f')}{P(f'')} > m$$

The above formula is simply Bayes' Theorem with the likelihood-ratio (middle term) expressed in terms of likelihoods instead of probabilities. The threshold,  $m$ , represents the burden of persuasion.<sup>164</sup> By rearranging terms, the above Bayesian probability test can be expressed as a special form of likelihood-ratio test:<sup>165</sup>

$$LR = \frac{L(f';e)}{L(f'';e)} > m \times \frac{P(f'')}{P(f')}$$

That is, a Bayesian or belief-based theory of fact-finding is equivalent to a specific form of likelihood reasoning in which the weight of evidence threshold depends on the fact-finder's prior beliefs. The probability, or belief-based, approach is like the proposed likelihood theory, but with a burden of persuasion defined as a fixed constant (the formal burden of persuasion) that gets scaled up (raising the burden of persuasion) or scaled down (lowering the burden of persuasion) to reflect the fact-finder's personal beliefs about the facts at the start of the case.

162. This presentation of posterior-probability reasoning in terms of likelihood ratios is similar to that of Kaplow, *supra* note 70, at 34–35.

163. Under the maintained assumption that there are only two possible factual theories, all of the various probability models discussed in Section III.A collapse into this same decision rule.

164. See sources cited *supra* notes 60–61 and accompanying text. This threshold,  $m$ , is not generally the same as the likelihood ratio threshold,  $k$ , discussed earlier in the paper.

165. This likelihood ratio test comes from the part of the above probability ratio test to the right of the equality: simply divide by the prior probability ratio to get the result.

The idea that a burden of persuasion would depend on the personal views of the fact-finder is troubling for at least two reasons. First, in contrast to some fields of study—where the influence of prior beliefs might be washed away by the collection of enough data—there is no reason to suspect that prior beliefs will be so diluted by the evidence in a typical fact-finding exercise.<sup>166</sup> Second, while there is nothing inherently debatable about how a fact-finder feels at the start of a case, the normative desirability of building these personal feelings into the fact-finding process is another matter altogether.<sup>167</sup> It seems fair to insist that the underlying epistemology of legal fact-finding should be a public concept, not a private one.<sup>168</sup> If this is too abstract, a concrete example illustrates the point.

The introduction of this paper briefly touched upon the recent Supreme Court case of *Peña-Rodriguez v. Colorado*. That case concerned a juror who explained during deliberations that the Latin-American defendant, on trial for sexual harassment, could be presumed guilty because, “in [this juror’s] experience as an ex-law enforcement officer, Mexican men had a bravado that caused them to believe they could do whatever they wanted with women.”<sup>169</sup> The juror went on to share his view that Mexican men were physically controlling of women and that “I think he did it because he’s Mexican and Mexican men take whatever they want.”<sup>170</sup> As if to dispel any doubt that these statements were meant to describe the prior probability of the defendant’s guilt, the juror concluded that “in his experience, ‘nine times out of ten Mexican men were guilty of being aggressive toward women and young girls.’”<sup>171</sup> Assume that these beliefs were sincerely held by the juror.

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166. See Leiter & Allen, *supra* note 15, at 1508 (“[I]ndividuals can begin from radically different perspectives, and each, in Bayesian terms, will be operating equally rationally. . . . In other contexts, such as science, these differences may be marginalized by convergence theorems that demonstrate that over time and with enough new evidence . . . result will [still] converge on the truth. There is nothing even remotely analogous to this in the condition of trials.”).

167. See PAWITAN, *supra* note 39, at 26 (“There is nothing really debatable about how one feels . . . [but] one’s formal action based on such feeling is open to genuine disagreement.”).

168. Cf. Taper & Lele, *supra* note 115, at 528 (“[Bayesian analysis] is held by many [to be] the most appropriate method of developing personal knowledge. This may be, but . . . [s]cience depends on a public epistemology not a private one.”).

169. *Peña-Rodriguez v. Colorado*, 137 S. Ct. 855, 862 (2017).

170. *Id.*

171. *Id.*

Any fact-finding process that is based on this fact-finder's personal beliefs about the facts necessarily starts from the fact-finder's prior beliefs about the facts—including prior beliefs that are, in turn, based on racial bias, gender stereotypes, assumptions about religious groups, and the like. A Bayesian understanding of fact-finding not only condones, but actually *requires* reliance on these personal prior beliefs.<sup>172</sup> In so doing, it prescribes the differential treatment of defendants. Applied to the situation in *Peña-Rodriguez*, a probability or belief-based approach to fact-finding would instruct the biased juror to consider only the evidence presented at trial, but to demand a smaller quantum of evidence in order to convict a Mexican defendant than to convict an otherwise identical defendant of some other race.

I reject that even the most ardent proponents of Bayesian analysis would support this implication. Nor do I believe that proponents of probability reasoning have much confidence in the use of prior beliefs in fact-finding.<sup>173</sup> The problem is that there is no way to avoid these implications and still retain a probability interpretation of the fact-finding process.

Cheng, for example, has attempted to argue that prior probability ratios always default to one in a fact-finding setting: “[T]he legal system imposes a constraint . . . [that normatively fixes] the prior odds ratio at 1 to start the plaintiff and the defendant in equipoise.”<sup>174</sup> But does this make sense? Cheng's justifications for the claim are that it equates to unbiased fact-finding,<sup>175</sup> is (somehow) implied by the plaintiff's satisfaction of the burden of production,<sup>176</sup> and is what fact-

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172. See Cheng, *supra* note 16, at 1267 (“In legal venues, one might fear that [prior probability ratios] embody prejudices against certain types of parties.”); cf. Posner, *supra* note 31, at 1514 (“Ideally we want the trier of fact to work from prior odds of 1 to 1 that the plaintiff or prosecutor has a meritorious case. A substantial departure from this position, in either direction, marks the trier of fact as biased.”).

173. *But cf.* Posner, *supra* note 31, at 1494–95 (“[If] the judge's prior odds are 100 to 1 in favor of guilt [and] the evidence creates a likelihood ratio of 8 to 1 that the defendant is not guilty, [then] the judge's posterior odds on guilt will still be 12.5 to 1. All this is perfectly rational.”).

174. Cheng, *supra* note 16, at 1273.

175. *Id.* at 1267 n.24 (citing Posner, *supra* note 31, at 1514).

176. *Id.* at 1267–68 (“As long as the plaintiff articulates a *prima facie* case and satisfies the burden of production, the case starts with both parties in equipoise.”). I fail to grasp the logic of this claim, but even if it were true, it would merely push the problem of the fact-finder's prior beliefs to the stage of determining whether the plaintiff has met the burden of production.

finders ought to do, even if they don't do it.<sup>177</sup> These arguments are not very inspiring, but even if they were, the use of uninformative or uniform prior probabilities would still not be the simple fix that it appears to be.

First—at a practical level—the argument confuses what society gets to control in the fact-finding setting. Fact-finders are not robots that can be programmed to have arbitrary prior beliefs. At least for now, all fact-finders are human. And while efforts at convincing fact-finders to adopt something closer to uniform prior beliefs may possibly be effective in some circumstances, in general we simply cannot ask a person for their personal beliefs after having seen the evidence, without the result being informed by their personal beliefs before seeing the evidence. That is, if the juror in *Peña-Rodriguez* sincerely believed the things he said in deliberations, then these prior beliefs will necessarily inform his posterior beliefs about the facts and thus the defendant's guilt. This is not a claim that this juror would intentionally try to subvert the legal process in order to act on his bias; it is simply a truism that if we ask this juror to state his personal beliefs about the facts given the evidence, he has no prior beliefs except his own to use in forming that opinion.

Second—at a technical level—uniform prior probabilities are not the reflection of initial-state ignorance that they appear to be. To say at the start of a case, “I do not have any idea whether the defendant is a murderer,” is obviously not the same as saying, “I believe there is a 50% probability that the defendant is a murderer.”<sup>178</sup> To further illustrate that point, suppose a defendant is on trial for two independent charges of murder, and that you know nothing about the defendant's guilt on either charge. Reflecting this initial-state ignorance by assigning a 50% prior probability of guilt on each event means—

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177. *Id.* at 1267 n.24. As a normative basis for this claim, Cheng and Pardo suggest that it would minimize the rate of fact-finding errors under certain distributional assumptions about the evidence-generating process. Edward K. Cheng & Michael S. Pardo, *Accuracy, Optimality and the Preponderance Standard*, 14 *LAW, PROBABILITY & RISK* 193 (2015).

178. See ROYALL, *supra* note 35, at 174 (“The reason why pure ignorance cannot be represented by a probability distribution is that every probability distribution represents a particular state of uncertain knowledge; none represents the absence of knowledge. . . . It is one thing to state that I do not know which of two possible values of  $\theta$  is true, or that I have no knowledge or no prior evidence about which is true. It is quite another to assert that the two values are equally probable.” (citations omitted)).

surprisingly—that you must believe there is a 75% probability that the defendant committed at least one of the two murders.<sup>179</sup> The attempt to represent initial-state ignorance with uniform prior probabilities in one aspect of the case translates into gratuitous information in another aspect of the case.<sup>180</sup>

The likelihood approach to fact-finding escapes all these problems. Likelihood reasoning minimizes the influence of a fact-finder's prior beliefs, prejudices, and biases, because it never asks the fact-finder to make any evaluation that involves them. Unlike Bayesian probability analysis, likelihood analysis never uses prior probabilities at all; it is reasoning from evidence alone. This is not to say that fact-finding modeled on likelihood analysis somehow debiases fact-finders or prevents them from ever acting on their beliefs and prejudices in the deliberative process. But by directing attention away from personal beliefs and toward the more objective question of the relative consistency of factual stories with the observed evidence, a likelihood approach to fact-finding at least aspires to minimize these influences. Put another way, likelihood analysis is the evidence-theory answer to the question Justice Sotomayor posed during oral arguments for *Peña-Rodriguez*, quoted in the first sentence of this paper: “Don’t we want deliberations on evidence and not deliberations on someone’s stereotypes and feelings about the race of a defendant?”<sup>181</sup> Of course we do. Likelihood reasoning encapsulates precisely this approach to fact-finding.

The proposed likelihood theory of fact-finding also eliminates the unsettling notion that different defendants should be facing different effective burdens of persuasion at the fact-finding stage of trial. In a likelihood approach, the idealized fact-finder is always comparing the same evidence to the same threshold, regardless of the fact-finder's personal feelings and convictions about the potential facts or parties to the case. This is not because likelihood analysis somehow imposes uniform prior probabilities in fact-finders' minds; it is because the likelihood approach never asks fact-finders to draw any opinion that

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179. This calculation assumes that the underlying events are stochastically independent, as suggested by the hypothetical. This is a simplifying assumption that can be relaxed without qualitatively changing the point of the example.

180. See EDWARDS, *supra* note 33, at 58.

181. Transcript of Oral Argument at 40:5–7, *Peña-Rodriguez v. Colorado*, 137 S. Ct. 855 (2017) (No. 15-606).

requires the use of these prior probabilities or beliefs in the first place. Theories of fact-finding under uniform prior probabilities are just another way in which the literature is grasping for something other than Bayesian analysis as the foundation for a coherent theory of fact-finding. As this paper has sought to show, likelihood reasoning fits the bill.

#### IV. LIMITATIONS

This is not to say that the likelihood theory of fact-finding is without limitations. Like any formal model, it is an abstraction that necessarily fails to capture much of the nuance and complexity of trial fact-finding. Some aspects of this abstraction are needed to provide the clarifying benefits of a formal model of the fact-finding process, but others could be relaxed in future work. The model could, for example, be extended to more explicitly consider the influence of self-interested advocacy on the production of evidence, and whether this holds any implications for fact-finder assessment of the likelihood function.<sup>182</sup>

Likelihood reasoning is also limited in that it operates within a frame of reference but cannot itself instruct the framing of a question.<sup>183</sup> This observation is itself an important implication of the model. It highlights a subject of possibly underappreciated importance.

To illustrate, one way in which the likelihood theory indicates that framing effects may significantly influence decision-making is in the granularity of fact-finding. In the earlier discussion of the disputed color of a stop light, for instance, it was assumed that the fact-finder considered the light to have three possible states—green, yellow, or red—and evaluated each of these three states individually in light of the evidence. An alternative assumption could be that the fact-finder considered the light to have two possible states—red or not-red—and evaluated each of these two states individually in light of the evidence. Nothing in the likelihood theory of fact-finding says which of these two ways to frame the fact question is correct or even preferable.

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182. See *supra* note 112 (discussing how the possibility of evidence tampering might be factored into the likelihood function). Cf. Sanchirico, *supra* note 11, at 205–07 (surveying contest models of strategic evidence production); *id.* at 223–33 (surveying omission models of strategic evidence production).

183. See *supra* note 147 (discussing ambiguity in the scope of fact-finding).

This is a limitation of the likelihood theory, but it is not a flaw of the model. Where the perfect fact-finder might ideally be able to identify the defendant's *exact* driving speed, the *exact* color of a stoplight, and the *exact* demands of prudence under the circumstances, flawed human fact-finders will inevitably treat theoretically continuous variables as discrete and group together alternatives that are not too conceptually separable.<sup>184</sup> The likelihood theory accommodates any granularity of possible facts, dictating only how the fact-finder should process the evidence within a given frame of reference.<sup>185</sup> But if changes in the granularity of the choice space might result in different fact-finding outcomes, then greater clarity on how the granularity of possible facts is and should be perceived by the fact-finder would strengthen the likelihood theory as a model of fact-finding.

Framing effects also matter under this likelihood theory of fact-finding in the different ways that an item of information could be seen to characterize irrelevant prior probability, or probative likelihood-relevant evidence, under different frames of reference. To illustrate, suppose a negligence case arises from a traffic collision. The defendant argues that she was driving carefully when her brakes randomly failed but produces no evidence of brake failure other than her own testimony. The plaintiff likewise produces no evidence on the specific condition of the defendant's brakes but instead replies with information on the rarity of random brake failures. This example is admittedly contrived—a more plausible focus of the plaintiff's response would be the lack of corroborating evidence of brake failure, if indeed none could be produced by the defendant—but it serves to illustrate an important point.

One view of the plaintiff's showing is that it describes the prior probability of the defendant's theory: random brake failures do not happen very often. But another view is that it helps the fact-finder contrast the probability of seeing the evidence of a traffic collision under the competing factual theories that the

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184. See Pardo & Allen, *supra* note 21, at 233–38 (discussing probable empirical determinants of how the fact-finder may perceive the granularity of the choice space).

185. This ability to accommodate composite states like “not red” would seem to violate the no-aggregation property of likelihood analysis. The reason that it doesn't is again a matter of framing: what likelihood analysis does not generally accommodate is the aggregation of alternatives in the perceived choice space. But this places no restriction on what the perceived choice space might be.

defendant was driving negligently (for which a crash seems relatively probable) and driving prudently (for which a crash from random brake failure seems relatively improbable). The choice of interpretation is driven by the framing of the question: is the fact-finder deciding whether the defendant was driving negligently (the second interpretation), or is the fact-finder deciding the specific question whether the defendant's brakes randomly failed in this instance (the first interpretation)? The likelihood theory does not itself indicate how this type of question should be framed, only how evidence should be processed within a given frame.

This uncertainty is uncomfortable and a limitation of the likelihood theory as developed, but it is again not a defect of the model. To repeat a prior point, so far as the difference between likelihood-relevant evidence and prior probability reflects the actual decision-making process of fact-finders, it is important that a theory of fact-finding reflect this distinction. Put another way, if framing effects are important in actual trial fact-finding, then it is only reasonable that they should enter into a theory of fact-finding as well. Future efforts to better understand what drives the way fact questions are and should be framed may strengthen this likelihood theory as a model of fact-finding.

Finally, while the proposed likelihood theory works well to explain the assessment of deliberate actions and historic facts—the type of facts that legal fact-finding is most often concerned with—it is an awkward fit to two other situations of practical importance. The first is a situation in which fact-finding encompasses something that is itself an aspect of a random process. If a dispute turns on the probability of injury resulting from equipment malfunction, for example, then information characterizing the probability of malfunction is not prior probability information but likelihood-relevant evidence of the rate of malfunction.<sup>186</sup> This does not represent a departure from likelihood reasoning, but it does blur the distinction between likelihood and probability analysis. The second is a situation in which the fact-finder is required to predict some future event in order to decide a case.<sup>187</sup> The need to predict,

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186. See *supra* note 157 (also interpreting habit evidence as a random process situation).

187. Under Section 7 of the Clayton Act, for example, courts are often asked to predict whether a proposed merger would have anticompetitive consequences. See

rather than discover historic facts, again blurs the distinction between likelihood and probability reasoning, and may yet leave room for Bayesian probability analysis in legal fact-finding.<sup>188</sup>

## CONCLUSION

In 1944, J. P. McBaine wrote simply, but powerfully, that then-existing confusion about the fact-finding process undermined the legal system:

[No legal system can] be praised for practicability if there exists vagueness, uncertainty or confusion as to the scope or extent of the burden [of persuasion], or if the language commonly employed to describe its scope or extent is not easily comprehensible to those whose duty it is to determine whether the burden has been sustained.<sup>189</sup>

If this is the test, then we must concede that, nearly seventy-five years later, our legal system *still* cannot be praised for practicability. McBaine, like many writers to follow, thought that probability concepts and a focus on the personal beliefs of fact-finders were the keys to untangling legal fact-finding. But as this article argues, probability and belief concepts are generally ill-suited to the task. I argue that to understand legal fact-finding and burdens of persuasion, our focus must be on the relative plausibility of competing narratives,<sup>190</sup> the relative weight of the evidence,<sup>191</sup> and the implications of likelihood reasoning.<sup>192</sup>

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15 U.S.C. § 18 (2016). Preventative injunctive remedies are another example of this type of predictive fact-finding. *See* OWEN M. FISS, THE CIVIL RIGHTS INJUNCTION 8–12 (1978) (distinguishing future-oriented preventative injunctions from retrospective reparative injunctions).

188. This is not to say that even in a predictive setting, probability or belief-based proof concepts are an obvious and comfortable fit. *Cf.* Uviller, *supra* note 24, at 848 (noting, in the context of character-based prediction of conduct that “[t]he law . . . has been particularly squeamish about predictive evidence; the result has been uncertain, inconsistent, and ill-defined rules”).

189. McBaine, *supra* note 10, at 242.

190. *See* Allen, *supra* note 18 (discussing empirical research on the cognitive process of fact-finders and the relative plausibility model of this process).

191. *See supra* Section II.B (explaining the use of likelihood ratios as way of understanding the meaning of the weight of evidence in a fact-finding context).

192. *See supra* Part III (highlighting several implications of the likelihood

The proposed likelihood theory of fact-finding breaks new ground on several fronts. It unifies understandings of the fact-finding process by providing a framework in which modern cognitive models of the fact-finding process extend to all burdens of persuasion. It also unifies the procedural theory of fact-finding with the law of evidence, aligning with evidence-law principles such as the ban on character reasoning<sup>193</sup> and fact-finder hostility to naked statistical evidence.<sup>194</sup> And it answers the unaddressed question in *Peña Rodriguez*, explaining the damage caused by fact-finder bias in all its forms—not just where racially motivated—and showing just how fundamentally bias, prejudice, and personal belief can infect deliberations if we are sloppy in how we understand and articulate burdens of persuasion to fact-finders.<sup>195</sup>

This is not to say that the likelihood theory of fact-finding is without limitations. Part IV of this paper notes several important limitations of the theory as presently developed. But the remaining challenges are narrow, and the benefits of this likelihood theory are real. Beyond the unifying and explanatory advantages already mentioned, the likelihood theory simplifies the cognitive model of fact-finding by showing that every burden of persuasion can be understood in terms of the same conceptual framework. That framework, in turn, is an intuitive and purely comparative approach of iteratively weighing evidence between competing factual theories. The result is a theory of legal fact-finding that is both internally consistent and in accord with common sense and actual trial practice.

To unlock the benefits of this new theory, we need to adjust the way we think about and describe fact-finding at trial. To be clear, I am not proposing that fact-finders should be instructed in the rigors of likelihood analysis. Nor am I proposing any semblance of the feared “trial by numbers” in which calculators replace pads and pencils.<sup>196</sup> And I also do not think that simply changing the language from *probability* to *likelihood* will change anything in practice. The distinction between these concepts is too esoteric for words alone to make a difference.

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theory of fact-finding, particularly where they contrast with those of extant theories).

193. See *supra* notes 156–157 and accompanying text.

194. See *supra* notes 158–161 and accompanying text.

195. See *supra* notes 1–7, 169–173 and accompanying text.

196. Calculators *with* pads and pencils, I support.

Rather, adjustments are needed to both our understanding and our description of the fact-finding process. For example, probability terms are now scattered throughout the law of evidence. The Federal Rules of Evidence define “relevant evidence” as anything having “any tendency to make a [material] fact more or less probable than it would be without the evidence.”<sup>197</sup> The Model Rules of Evidence define “finding a fact” as “determining that [the fact’s] existence is more probable than its non-existence.”<sup>198</sup> Jury instructions now commonly define burdens of persuasion in probability terms.<sup>199</sup> And some scholars have gone so far as to say that “a lawsuit is essentially a search for probabilities.”<sup>200</sup> A predictable response is that these terms are not intended to be interpreted in a technically accurate sense. But that only solidifies the importance of understanding what the terms could, and do, mean in legal fact-finding.

As mentioned in the introduction of this paper, Richard Royall provides a helpful way to conceptualize the options:<sup>201</sup>

- Bayesian posterior probability answers the question: “What do I believe, now that I have seen this evidence?”
- Likelihood answers the more basic question: “What does this evidence show?”

That is, Bayesian probability is fundamentally a description of belief; likelihood is fundamentally a description of weight of evidence.<sup>202</sup>

Over the years, many scholars have written that fact-finders’ beliefs are the thing we care about, not some abstract concept of the weight of the evidence.<sup>203</sup> But this article shows the

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197. FED. R. EVID. 401.

198. MODEL CODE OF EVIDENCE Rule 1(5) (AM. LAW INST. 1942) (internal quotation marks omitted).

199. See *supra* note 50 and accompanying text.

200. MCCORMICK ON EVIDENCE, *supra* note 57, at 669.

201. See sources cited *supra* note 23 and accompanying text.

202. Royall, *supra* note 23, at 127–31.

203. *E.g.* James, *supra* note 123, at 53 (“All would agree that what counts is the jury’s belief in the existence (or non-existence) of the disputed fact, and the extent to which the evidence actually produces that belief; surely we are not seeking the jury’s estimate of the weight of evidence in the abstract . . .”); McBaine, *supra* note 10, at 247 (“The [common jury instruction speaks of] the weight of the evidence. It does not, as it should do, direct [attention] to the degree of belief which the proponent of the proposition must produce . . . before he is entitled to a finding favorable to him.”).

exact opposite to be true. Most fact-finding makes sense only when it is approached as a question of weight of evidence alone.<sup>204</sup> Not only do we not care about fact-finders' personal beliefs in fact-finding, but the role of personal beliefs should be minimized and extracted from fact-finding as far as it can be.

The direct contribution of this new theory of fact-finding is a clearer articulation of the ideal fact-finding process. It remains to future work to contrast this platonic model with practice and procedure, and to find areas for improvement of the status quo. But, already, several areas for improvement are evident. First, many common articulations of the burdens of persuasion err by instructing the fact-finder to consider the probability of individual facts in isolation. As shown here, fact-finders should be focused on relative plausibility, not propositional probability, and should be thinking about comprehensive factual stories, not individual facts in artificial isolation. Second, common articulations of the burdens of persuasion err by instructing the fact-finder to act from personal beliefs about the facts. While fact-finders should, as ever, use their experience and intuition to guide their deliberations, their beliefs are not the lodestar of this exercise, and should actually be suppressed as far as possible in the endeavor. The task of the fact-finder is not to form personal beliefs about the facts but is instead to weigh the evidence produced at trial, comparing the relative plausibility of factual stories on the observed evidence in deciding the outcome of a case. This is no less true in criminal cases under the reasonable doubt standard than it is in civil cases under the preponderance standard. Third, to the extent that any information merely describes prior probabilities, it should be excluded from the evidence as irrelevant. To do so in a systematic way will require a clearer understanding of how fact-finding questions should be framed at trial. This is no small task, but one that should not be shirked given the lives and livelihoods at stake.

In some respects, these suggestions for improvement simply codify or extend what the trial process already does.

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204. See SUSAN HAACK, EVIDENCE MATTERS: SCIENCE, PROOF, AND TRUTH IN THE LAW 51–53 (William Twining, Christopher McCrudden & Bronwen Morgan eds., 2014) (expressing the concurrent view that the epistemic language of burdens of persuasion and jury instructions often reveals a primary concern with the objective strength of evidence in a case, as opposed to the subjective beliefs of the fact-finder).

Many fact-finders *already* operate as the likelihood theory requires—refusing to assign liability on purely statistical evidence, for example.<sup>205</sup> Rules of evidence like the prohibition on character reasoning *already* echo a concern with the effect of the irrelevance of prior probabilities in fact-finding.<sup>206</sup> And the modest protections of voir dire and precautionary jury instructions *already* dilute the influence of prior beliefs and fact-finder bias in some cases.<sup>207</sup>

But fortuitous success is weak assurance against future mistake.<sup>208</sup> And these diverse rules and practices are no substitute for clarity about the underlying theory of fact-finding and for systematic efforts to approach that ideal.<sup>209</sup> The likelihood theory of fact-finding supplies this clarity and this ideal, showing how all legal fact-finding fits within the unifying framework of a guided search for the most likely story.

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205. See *supra* notes 154–154, 158–161 and accompanying text.

206. See *supra* notes 156–156 and accompanying text.

207. See, e.g., FED. R. CIV. P. 47 (selecting jurors in federal civil trials); FED. R. CRIM. P. 24 (selecting jurors in federal criminal trials); FED. R. EVID. 105 (limiting the scope of evidence by instruction to the jury in federal trial); SEVENTH CIRCUIT FEDERAL CIVIL JURY INSTRUCTIONS, *supra* note 8 § 1.06 (instructing the jury on what is *not* evidence).

208. See Learned Hand, *Have the Bench and Bar Anything to Contribute to the Teaching of Law?*, 24 MICH. L. REV. 466, 470 (1926) (“[A] right result reached by unsound reasons gives no assurance of permanent acquisition.”).

209. Lempert, *supra* note 30, at 1056 (“Whatever enables lawyers to think more clearly is of practical importance.”).